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利用後向特徵值快速求解隔熱壁之溫度分佈

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摘要

於本論文中，後向特徵值法將被推導並運用於快速求解隔熱壁之溫度分佈。其中安置於內層壁的薄片金屬將作為熱通量的照射面，而其他熱絕緣的材料則依序緊密地排列成多層壁;至於各層接觸面間所設定之獨立坐標系統則用來建構多層壁熱傳之特徵方程式。依據熱連續及熱通量守恆的原理，相關特徵參數值將靈活地給定;亦即內部各層之特徵參數值柔性地設為π，而滿足邊界條件之非線性特徵參數則歸結於最內與最外層壁;經後向遞迴程序計算後，非線性的特徵值即可順利地解出;而以往求解程過中產生的非線性疊代困擾於此將可完全的排除。於流明強度 900~1250 lux 照射下，理論計算值解與實驗結果呈現良好的一致性(其最大誤差不超過 15%);換句話說，以更快之計算速度而獲致整體的熱行為模式於此將獲得印證。

關鍵詞：特徵值,絕熱材料, 後向特徵值法,遞迴法。

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A Feasible Backward-eigenvalue Method to Speedily Access the Thermal Profile on Heat-insulating Multilayer

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Abstract

In this study, a feasible backward-eigenvalue method is introduced to analyze the temperature distribution of heat-insulating multilayer. In which, a thin-metal sheet, mounted as an inner layer, is subjected to heat flux, and separable thermal insulating material ,in the multilayer, is then well compacted subsequently while independent coordinate specified on the each interface of slabs will be used to construct the governing eigen-equations of heat conduction for multilayers. Base on thermal continuity law including temperature as well as heat flux, each eigen-parameter accessed from the corresponding exterior slab might be feasibly set as π, and those at boundary surface could be left to be constrained by boundary conditions. Thus the difficulty encountered in previous method, arisen from iteratively solving nonlinear eigen- function for each slab, could be fully avoided by leaving these troubles on the first layer and which might be easily solved using backward – eigenvalues recurrence. Compared with analytic and experimental results accessed from the irradiation of illumination falling within 900~1250 lux, both are found to behave a good agreement and their maximum relative error will be less than 20% ,i.e., the faster calculating speed to approach overall thermal behaviour has been confirmed in this study.

Keywords: Eigenvalue, Thermal insulating material, Backward-eigenvalue method, Recurrence

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1. Introduction

To analytically study the heat-transient problem, individual eigenvalue determined from corresponding equation seems to be inevitable in advance. Indeed, the calculating procedure for nonlinear eigen-roots is so tedious that it usually makes the author hard to be accessed (James et al. 1990；Burniston and Sliewert, 1973). To improve above inadequacy, a numerical method computing these roots of transcendental equations were developed (Ozisik, 1979). Here the proposed analytic model provides a workable manner to carry out the eigen-calculation, but the time -saving, the solutions iteratively solved from nonlinear equation, seems to be not so notable. Recently, CFD commercial package (computational fluid design) , with its special advantage in quick simulation, has attracted public interest and widely used in massive calculation. While couple with numerical skill and finite element theory, 3-D partial equations of heat conduction & convective model could be discretized into a set of simultaneous algebraic equations which might be well formulated into a corresponding tridiagonal matrix ,i.e., the troubled eigenvalue problem could be escaped. Base on above benefit, Wang and Xu (Wang and Xu, 2006；Xu and Wang, 2008) constructed an analogues R-C thermal network to undergo an indoor -ventilation simulation with Genetic algorithm (GA). Due to the duplicated implicit operation for R-C thermal network could not be further compressed, thus much time to have results is still another problem. Besides, Himanshu (Dehra, 2009) utilized a source -matrices method to study the heat transport on photovoltaic solar wall. Although the computational time has been significantly shortened, such a manipulation is found to be only validated for steady consideration, and material specific heat comparable to capacitors in thermal network should be taken into account if transient heat is considered. To strengthen above performance in comprehensive calculation, Liu et al. (2011) established a modified star-type R-C thermal network for concrete core cooling design. Here additional simultaneous equations were also required to enclosure the analytic model and finer grids over the whole network, to toward exact solution, was then constructed by Kontoleon (Kontoleon, 2011). Even though such prearrangement, the calculated process to have simulated results still became dull and inefficient. Generalizing from previous study by means of computational technique, the trouble of determining nonlinear eigenvalue could be ignored. However, the accumulation of truncation errors, round off errors as well as time-consuming seems to be out-of-control and left to be desired. Indeed, most thermal model for multilayers in engineering application might be further simplified by way of scalar analysis. Such analytic method using backward-eigenvalues to speed up the results, a modified manner from Ho. et al. (2013) in forward-formulation, has attracted our interest and become the main objective of this study.

2. Analysis

Prior to embark on analytic analysis, 1-D heat conduction model with composite medium composed by parallel layer of slabs is illustrated in Fig.1. Here, the left surface of thin-metal sheet, as the first slab, is subjected to a constant heat flux q and individual thermal insulated material is then specified in the following layers successively. During the whole experimental or analytic process, thermal continuity is assumed to prevail within the whole working region.

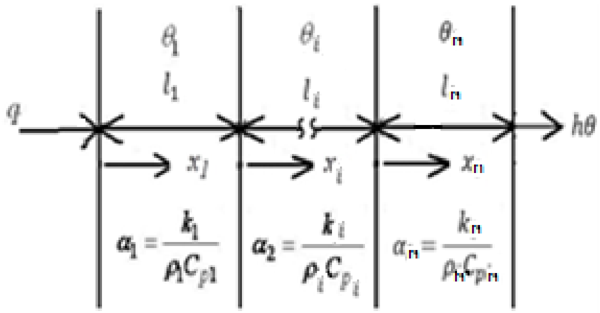

Figure 1. Geometric structure of composite layers for 1-D heat conduction

2.1 Assumptions

1. Thermal property of working material including density, thermal conductivity, specific heat as well as thermal diffusivity is considered to be constant& isotropic.
2. Base on scalar analysis, heat will be primarily conducted in x-direction conduction while the thickness of slab compared to its length & width is smaller enough. Thus 1-D heat conduction model is adequate to meet the demand of simulation.
3. Contacted thermal resistance or thermal dissipation at the interface might be neglected by taking the closely compacted slabs into account.

2.2 Governing equations

Coupling with above assumptions, a partial differentiation equation governing 1-D heat conduction is given in Eqs.(1). Here relevant boundary and initial conditions are enclosed in Eqs.(2)~Eqs.(4) and Eqs.(5) respectively.

$$\frac{\partial \theta}{\partial t} = \alpha \frac{\partial^2 \theta}{\partial x^2} \tag{1}$$

$$q = -k_1 \frac{\partial \theta_1}{\partial x_1} \qquad x_1 = 0 \tag{2}$$

$$\theta_i = \theta_{i-1}$$
$$k_i \frac{\partial \theta_i}{\partial x_i} = k_{i-1} \frac{\partial \theta_{i-1}}{\partial x_{i-1}} \qquad x_i = \ell i \qquad \text{for} \quad i = 2 \sim N\text{-}1 \tag{3}$$

$$k_N \frac{\partial \theta_N}{\partial x_N} = -h\theta_N \qquad x_N = \ell_N \tag{4}$$

$$\theta_i = 0. \qquad \text{for} \quad i = 1 \sim N \quad \text{at} \quad t = 0 \tag{5}$$

Thermal property k_i , α_i, ,$\theta_i = T_i - T_\infty$ and h individually indicates material heat conductivity, thermal diffusivity, temperature distribution of i-th layer and convective coefficient at right boundary surface. Symbol q designates the heat flux imposed at left surface and then x_i is the coordinate measured from the interface of *i-1* and *i* th layer.

2.3 Analytic Solution

Invoking separation method, analytic solution of Eqs.(1) could be sucessfully carried out in Eqs.(6) where steady temperature θo in Eqs.(7) easily estimated from anaoglous thermal network illustrated is given in Fig. 2.

$$\theta_1 = e^{-\beta_i^2 t}\left(A_1 \cos\frac{\beta_1}{\sqrt{\alpha_1}}x_1\right) + \theta_0 - \frac{q}{k_1}x_1$$

$$\theta_i = e^{-\beta_i^2 t}\left(A_i \cos\frac{\beta_i}{\sqrt{\alpha_i}}x_i + B_i \sin\frac{\beta_i}{\sqrt{\alpha_i}}x_i\right)\theta_0 - q\left[\sum_{i=0}^{i-1}\left(\frac{l_i}{k_i}\right) + \frac{x_i}{k_i}\right] \quad \text{for i =2,n-1} \qquad (6)$$

$$\theta_o = q*[\ \Sigma l_i/k_i+(1/h)] \qquad (7)$$

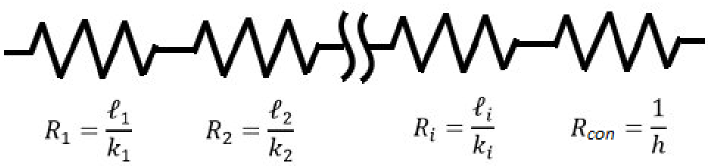

Figure 2. Sketch of thermal network for multi-layered conduction

Subjected to boundary conditions of Eqs.(2)~(4), Eqs.(8)~Eqs.(10) provide recurred relation of coefficients A_i and B_i . In which, the equation governing thermal continuity at the interface of 1-th and 2- th layer is yielded in Eqs.(10), and x=-1 at Eqs.(8) satisfies the adiabatic condition at right boundary surface as heat convective coefficient h is assigned to be zero. Base on above arrangement, a trial and error metod solving these nonliear eigen-equations becomes a really hard work to be accessed unless a firther simplification is under consideration.

$$\frac{B_N}{A_N} = x \qquad x_N = l_N \qquad (8)$$

$$\frac{B_i}{A_i} = -\frac{k_i\sqrt{\alpha_{i+1}}\beta_{i+1}\frac{A_{i+1}}{B_{i+1}} + k_{i+1}\sqrt{\alpha_i}\beta_i \cot\frac{\beta_i l_i}{\sqrt{\alpha_i}}}{k_{i+1}\sqrt{\alpha_i}\beta_i - k_i\sqrt{\alpha_{i+1}}\beta_{i+1}\frac{A_{i+1}}{B_{i+1}}\cot\frac{\beta_i l_i}{\sqrt{\alpha_i}}} \qquad x_i = l_{N-1} \sim l_2 \qquad (9)$$

$$\frac{B_2}{A_2} = \frac{k_1\sqrt{\alpha_2}}{k_2\sqrt{\alpha_1}}\tan\left(\frac{\beta_1 l_1}{\sqrt{\alpha_1}}\right) \qquad x_1 = l_1 \tag{10}$$

To overcome above difficulty, a flexible eigenvalue method in backward manner is then developed ,that is, let each argument of cotangent function ,in Eqs.(9), be equal to π and a simplified form concerning the relation of A_i/B_i could be resulted in Eq.(11) using the regressive reccurence of Eqs.(8)~Eqs.(9). By way of the equivalence of Eqs.(10) and Eqs.(11) provided i is back numbered from N to 2, β_1 could be succesfully generalized while a thin-metal sheet ,as the first slab, is taken into account. Thus nonliear troubles emergying at forgoing eigenvalue-problem might be fully escaped and simple relation of A_i and B_i as well as individual eigenvalue is explictly expressed in Eqs.(11)~Eqs.(12).

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$$\frac{B_i}{A_i} = -\frac{k_{i+1}\beta_{i+1}\sqrt{\alpha_i}}{k_i\beta_i\sqrt{\alpha_{i+1}}} = -\frac{k_N l_i}{k_i l_N} \tag{11}$$

where

$$\beta_1 = \sqrt{\frac{2\pi\alpha_1 k_N}{l_1 l_N k_1}} \qquad \beta_i = \frac{\sqrt{\alpha_i}\,\pi}{l_i} \tag{12}$$

Before we complete the analytic solution, the determination of coefficients A_i and B_i is still required using Fourier’s expansion to meet initial condition in Eqs.(5) ,and resultant formula might be readily obtainable in Eqs.(13).

$$A_1 = \frac{q l_1}{k_1} - \theta_0 \qquad A_i = \frac{-4q l_i}{k_i} \tag{13}$$

3. Results and discussion

Unlike the feasible backward-eigenvalue method proposed in this study, the difficulty encountered in previous research, based on the orthogonally of nonlinear eigen-fuction to solve unknown coefficients A_i, and B_i, might be absent. Here the value of π rated to the argument of triangular functions ,in Eqs.(9), not only still holds periodic characteristic, but also makes individual eigen-value be easily predicted from the explicit relation of A_i, and B_i developed. That is quite different from implicit iteration needed in forgoing model. Furthermore, the value of ration x assigned in Eqs.(8) is fully dependent on the boundary condition specified at right wall i.e., x=-1 indicates the adiabatic environment or x=1 is given in full convection. That also provides special advantages to determine coefficients A_n & B_n for thermal insulation considered here ,and also makes β_1 determined becomes possible by the backward recurrence despite of the β_n selected.

Subsequently, our discussion will forward to compare the experimental and analytic results accessed. In Fig.3, both temperature distributions are found to be in close agreement under the experiment with multi- layers closely compacted by a thin stainless plate and thermal proof material. As the light flux 900 lux is illuminated at the surface of thin metal plate positioned as an inner layer, both analytic and experimental surficial temperature T1, measured at left boundary exposed to heat flux, are found to steadily grow up to 45^o C within the period of about 30 min and their maximum relative error of 12% could be under control. As to the surficial temperature of fiber T2 at right boundary ambient to environment, both exhibit an excellent consistence and nearly keep at 18^oC. Thus the thermal insulation could be effectively performed by the usage of thermal proof fiber. Similar to above consistent trend, Fig.4 displays the temperature dissemination resulted from the stronger irradiation of 1250 lux. Here the time required for T1 quickly warmed up to 50 ^{o}C is only 7 min while T2 is then experienced at a smaller increase of 25 ^{o}C ~30 ^{o}C. And their relative errors are found to be less than 10% & 15 % delivered at T1 & T2 respectively. Such the shorter time needed to approach the saturated state seems not so surprised due to the thermal energy will be believed to be quickly accumulated at the metal plate.

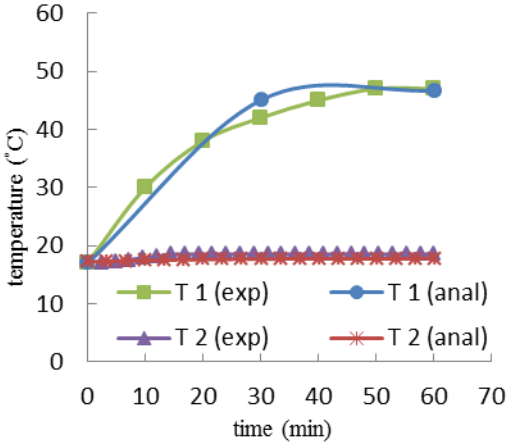

Figure 3. Comparisons of analytic & experimental temperature history at T1 and T2 for double- layer of fe –fiber slabs irradiated by 900 lux

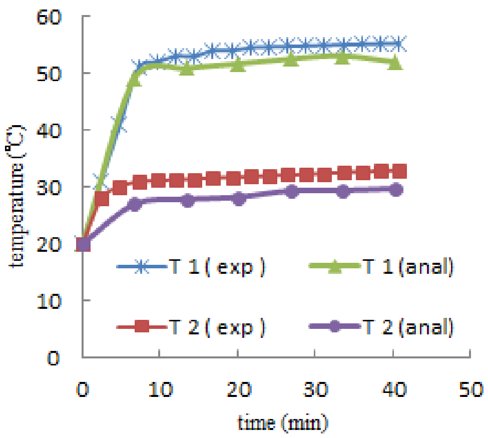

Figure 4. Comparisons of analytic & experimental temperature history at T1 and T2 for double layers of fe –fiber slabs irradiated by 1250 lux

So far our discussion has been focused on the working temperature varying with duration for different illumination on double layers, next our attention will turn to understand how the temperature profile changes by compacting a wooden plate additionally. Through it, both experimental and analytic outcome, illustrated in Fig.5, tell that about 10 min is required for T1 to reach at saturated temperature 55 ^{o}C and which is slightly delay compared to 7 min measured from the case of double-layers. Also the temperature at outer surface adjacent to environment, T3, nearly keeps at about 22 ^{o}C closer to the ambient temperature 20 ^{o}C. That means the heat flow could be effectively retarded by the mid- layer of fiber, and added compacted wooden sheet seems to be unnecessary. That could be identified by the layout of Fig.5 where both analytic and experimental results for T1 is distributed in excellent agreement and about 20%

discrepancy ,however, appears at both distributions of T3.

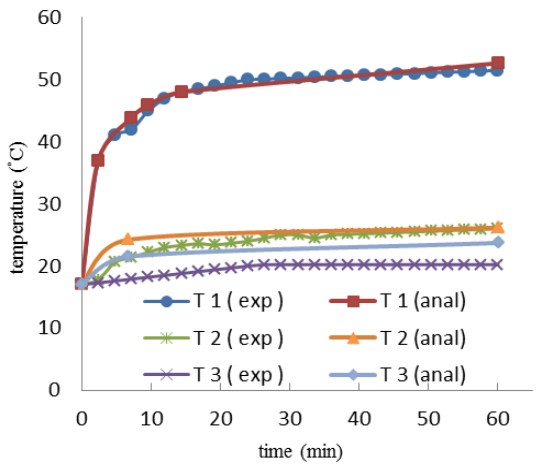

Figure 5. Comparisons of analytic & experimental temperature history at T1, T2 and T3 for multi-layer of fe–fiber-wood irradiated by 1250 lux.

4. Conclusion

Summary from above discussion, the manipulation of flexible backward-eigenvalue, proposed in this article, has been proven to possess special advantage over traditional methods. Here analytic accuracy compatible to experimental results has been confirmed and calculating speed, base on each eigenvalue explicitly determined by backward recurrence, could be significantly promoted by setting eigen-argument to be π. Although the calculating rate is primarily dependent on the individual eigenvalue specified, the final solution will approach to be unique. By way of the innovative backward-manner introduced in this study, the tedious operating process using commercial package or numerical skill could be fully avoided, i.e., the time-saving ,without losing the global thermal behavior, benefited from backward- eigenvalue manner proposed could be identified. Thus further extending present technology to the future research on thermal transmission of 2-D or 3-D composite-wall might be more desired.

5. Nomenclature

A_i the coefficient of i-th layered eigen-cosin function

A_N the coefficient of N-th layered eigen-cosin function

B_i the coefficient of i-th layered eigen-sin function

B_N the coefficient of N-th layered eigen-sin function

T_i the temperature of i-th layer [K]

T_∞ ambient temperature [K]

h convective coefficient [$Wm^{-2}K^{-1}$]

k_i thermal conductivity of of i-th layer [$Wm^{-1}K^{-1}$]

k_N thermal conductivity of of N-th layer [$Wm^{-1}K^{-1}$]

ℓ_i the thickness of i-th layer [m]

ℓ_N the thickness of N-th layer [m]

q incident energy density [W/m^2]

t time duration [s]

x_i the transversal coordinate of i-th layer [m]

x_N the transversal coordinate of N-th layer [m]

α_i thermal diffusivity of of i-th layer [m^2/s]

β_i the eigenvalue of of i-th layer

θ_i T_i-T_∞ i- th layered temperature relative to ambient [K]

θ_N T_N-T_∞ N- th layered temperature relative to ambient [K]

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