National Ilan University

國立宜蘭大學工程學刊第 11 期，35-45 頁，2015 年 12 月
Bulletin of College of Engineering National Ilan University No. 11, PP. 35-45, December 2015

Bulletin of College of Engineering
http://engineering.niu.edu.tw/main.php

Investigate critical characteristic of harmonic ferro-dynamics subjected to the induction of uniform magnetic field

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Abstract

The critical characteristic of ferro-dynamics, subjected to harmonic oscillation, is investigated in this study. Here an quasic-coordinate transformation simplifying the model proposed will be undertaken by transferring inertia coordinate to self-designed oscillating system. Thus the analytic solution carried out will become easier using the separation method. Through experimental observation and analytic analysis, the time-varying ferro- characteristic including surface elevation $\boldsymbol{\delta}$" so called progressive fluctuating amplitude", surficial fluctuating velocity v' and relative velocity u', will be quantified. Consequently, the magnitude of $\boldsymbol{\delta}$, ***v'*** is individually found to be an order of square and cubic $\boldsymbol{\omega}$, and then ***u'*** behaves a linear relation of $\boldsymbol{\omega}$. Besides, the nodal position of ferro-wave, under the enforcement of uniform magnetic field, could be successfully predicted at the middle of moving system and which might be forwarded to the purpose of shock-absorption. While compare the consequence of $\boldsymbol{\delta}$ accessed from theoretic analysis and experiment, both are found to be consistent, 0~0.38mm, within the oscillating frequency $\boldsymbol{\omega}$<1.2 rps subjected to field intensity B=0.2T. Here their relative error, less than 10 %, will be experienced. However, the remarkable departure for $\boldsymbol{\delta}$, 2.5 mm (theory) and 1.3mm (experiment), begin to emerge as the oscillating frequency $\boldsymbol{\omega}$ is extended to 2.4rps.

Keywords: humping ferro-damp, nodal prediction, shockproof

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1. Introduction

Due to the reversal characteristic without magnetic hysteresis, ferro-fluid, known as an intelligent fluid used in quake-buffer, has gradually attracts public attention. Prior to the application for industrial purpose, the understanding of critical ferro-property, induced by field intensity, seems to be inevitable. In 1969, a ferro-theory dealing with apparent viscosity of water-base ferro-sample was initiated (Hall, W. F. and Busenberg, 1969), where the relative error of thermal property, without taking thermal agitation and Brownian motion into account, is found to be up to one or two orders of magnitude. To correct above deficit, a modified Langevin theory together with the degree freedom of ferro-particle as well as directional field was proposed (R.E. Rosensweig, 1985). Here the multi-correlation of ferro-viscosity on asymmetric stress was still left to be desired. Recently, micro-technology, A.C chip embedded in the bio-circuit to detect ferro-signal, has been widely discussed for biomedical research (Zahn, M. and L.L. Pioch, 1985; A. Zcurer, R. Richter and L. Rchberg, 1999; M. Zhan, 2001). Yet, the prompt access of damping response will become hard to be accessed due to the non-synchronization exists between ferro- magnetization and A.C frequency. To improve the lagging phenomena mentioned above, the performance of MEMS (micro electromechanical system) technology (Nicole Pamme, 2006) could not only achieve the sampling accurateness, but the data -resolution might be significantly improved. Conversely, such costly testing instrument with tedious experimental process will be unaffordable for local laboratory. Overview from previous disadvantages, a modified oscillating device based on the Rayleigh-Taylor theory (J. E. Ho, 2014; J. E. Ho, C. L. Yen, and J. X. Lin, 2015) is set up in this study. Here critical ferro-characteristic on the periodic resonance of ferro-surface, developed from theoretic model and empirical method, will be clearly discussed. Prior to the further application for industrial purpose, how to establish the relationship among surface elevation, surficial fluctuating velocity, relative velocity, oscillating frequency and nodal position, under the enforcement of field intensity, has arisen our interest and becomes the main objective in this article.

2. Analysis

To successfully formulate an empirical method, several reasonable assumptions without losing overall behavior should be made in advance.

2.1 Assumptions

(1) Both Brownian effect, arisen from thermal effect, and collinear interaction of magnetized particles are small compared to periodic force applied, thus their influence on ferro-motion might be ignored.

(2) Ferro-property of micro-scale ferro-particle, induced inside the test sample, is considered to be isotropic and uniform distribution.

(3) The practical magnetization M of ferro-solution, based on Langevin's theory [1], is assumed to be saturated for the strong magnetic field, greater than 0.1T, enforced in this study.

(4) Since the ferro-depth of testing solution is much less than the length covered, the absence of flow momentum in vertical component will not lose the global behavior.

(5) Just for the constant magnetic pressure, induced vertically, is manipulated, ferro-dynamics pressure ***p*** assumed to be function of ***x*** might be reasonable.

2.2 Governing equations

Initially, a ferrohydrodynamics(FHD) equation governing periodic ferro-motion, with the absence of transversal ferro-velocity, is introduced in Eqs.(1). Here symbol ***u*** indicates the oscillating ferro-velocity along the ***x*** direction. To effectively analyze the equation mentioned above, Eqs.(2) offers a series of relations to undergo a coordinate transformation, i.e., transfer the inertial coordinate (x,t) to relative coordinate (x',τ) subjected to the slider. Consequently, a quasi-moving recycling model in Eqs.(3), after substituting Eqs.(2) into Eqs.(1), might be detail developed. Here ***U, Uo*** is individually defined as the periodic velocity and the maximum velocity of slider, $\boldsymbol{u}'$ yields the ferro- velocity relative to the velocity of slider ***U***, ***ρ*** is the density of ferro-solution, ***p*** indicates the ferro-dynamics pressure and μ is then given as ferro-viscosity.

$$\left[\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+v'\frac{\partial u}{\partial y}\right]=-\frac{1}{\rho}\frac{\partial p}{\partial x}+v\left[\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}\right] \qquad v=\frac{\mu}{\rho} \tag{1}$$

$$u=U+u'U=U_0e^{iwt}\frac{\partial}{\partial t}=\frac{\partial}{\partial \tau}-U\frac{\partial}{\partial x'}x=x'+\int Udt \tag{2}$$

$$\rho\left[\frac{\partial(U+u)}{\partial \tau}-U\frac{\partial(U+u)}{\partial x}+U\frac{\partial(U+u)}{\partial x}+u\frac{\partial(U+u)}{\partial x}+v'\frac{\partial(U+u)}{\partial y}\right]=\frac{-\partial p}{\partial x}+\mu\left[\frac{\partial^2(U+u')}{\partial x^2}+\frac{\partial^2(U+u')}{\partial y^2}\right] \tag{3}$$

While rearrange Eqs.(3) together with the zero value weighed by ***U*** differentiated respect to x and y individually, an equilibrant equation, in Eqs.(4), will be carried out. Here the value of R_e (Reynold number), base on scale- analysis skill, is roughly estimated about 50, namely, the convective terms is found to be smaller enough compared to viscous diffusion. Thus the Eqs.(4) might be further simplified and resulted in Eqs.(5).

$$\left[\frac{\partial u'}{\partial \tau}+u\frac{\partial u'}{\partial x}+v'\frac{\partial u'}{\partial y}\right]=-\frac{1}{\rho}\frac{\partial p}{\partial x}+v\left[\frac{\partial^2 u'}{\partial x^2}+\frac{\partial^2 u'}{\partial y^2}\right] \qquad R_e=\frac{\rho U_o \ell}{\mu} \tag{4}$$

$$\frac{\partial u'}{\partial \tau}=-\frac{1}{\rho}\frac{-\partial p}{\partial x}+v\frac{\partial^2 u'}{\partial x^2} \tag{5}$$

where ℓ is the length of cylindrical vessel containing ferro-sample.

Next our focus will be concentrated on how to solve the Eqs.(5) in which the determination of pressure term is still required prior to carry out the exact solution. While employ the gradient of Navier- Stroke equation, a Laplace's equation related to ferro-pressure, as shown in Eqs.(6), is developed out with the boundary conditions specified in Eqs.(7). Here a harmonic function of ***p***(x,t), conveyed to satisfy above equations, could be taken as a particular solution approximately equivalent to the magnetic pressure induced, and then δ, the levitation of ferro-surface stated in Eqs.(8), might be directly developed. Here ***r***, ω mean the recycling radius and angular velocity, g and gm are the gravity, induced magnetic gravity acceleration.

$$\nabla^2 p = o \qquad -\frac{\partial p}{\partial x}\Big|_{x=-\frac{\ell}{2},\ t=0} = \rho r\omega^2 \qquad u'(x, t=0) = 0 \qquad u'(-\ell/_2, t) = u'(\ell/_2, t) = 0 \tag{6}$$

$$p = A_0 e^{i(\omega t + \frac{\pi}{\ell}x)} = \rho\left(g + g_m\right)\delta \qquad A_0 = \frac{\rho r\omega^2 \ell}{\pi} \tag{7}$$

$$\delta = \frac{r\omega^2 \ell}{\pi(g + g_m)} cos\ (\omega t) sin\ (\frac{\pi}{\ell}x) \tag{8}$$

To successfully evaluate the vertical velocity of ferro-surface $\boldsymbol{v}'$, an additional kinematic condition of ferro-surface, and the resultant profile, after the further integration, might be both presented in Eqs.(9). Coupling with Eqs.(4)~(5),the analytic velocity $\boldsymbol{u}'$ in Eqs.(10) could be also accessed while the separation method is commenced.

$$\frac{\partial \delta}{\partial t} = v' \qquad v' = -\frac{r\omega^3 \ell}{\pi(g + g_m)} sin\ (\omega t) sin\ (\frac{\pi}{\ell}x) \tag{9}$$

$$u' = \frac{r\omega}{\sqrt{1 + (\frac{v}{\omega})^2 (\frac{\pi}{\ell})^4}}\left[sin\left(\omega t + \frac{\pi}{\ell}x - \theta\right) - \frac{2x}{\ell} cos\ (\omega t - \theta)\right] \tag{10}$$

2.3 Experimental procedure

To understand the critical characteristic of harmonic ferro-dynamics, a self-design periodic oscillating system consisting of DC power supply, video camera, slider and ferro-sample etc., in Fig.1, is set up, which features as special advantages of smaller size, economical utility and easy-to-use. Here the half ferro–wavelength, to study the critical characteristic from the maximum resonance, will be created by confining the length of sample container $\boldsymbol{\ell}$=0.02m and recycling radius $\boldsymbol{r}$=0.015m. Moreover, an empirical apparent gravity $\boldsymbol{gm}$=4 m/s^2 ,under the enforcement of field intensity $\boldsymbol{B}$=0.2 T, is then estimated from the weight –increment of ferro-solution containing volumetric concentration $\boldsymbol{\phi}$=0.4 at 25℃,and which, based on Langevin's theory, also induces the kinematical viscosity $\boldsymbol{\nu}$= 0.0001 m2/s. While the voltage 0 ~ 10V is regulated by turns, corresponding periodic frequency of slider motion, 0 ~ 2.4 rps, might be initiated by DC motor and the fluctuating ferro-surface on traveling wave ,illustrated in Fig.2, could be promptly captured by video camera if the stable oscillation is in progress.

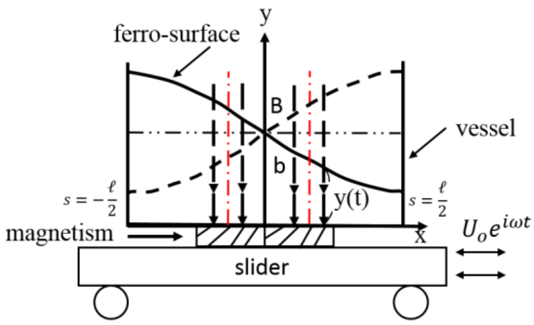

Figure 1. The sketch of periodic oscillating device

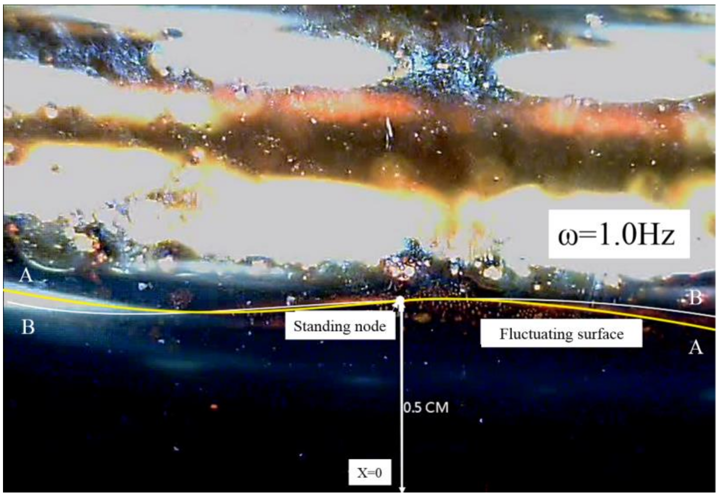

Figure 2. The photograph of fluctuating ferro-surface

3. Results and discussion

Initially, let’s examine the time-varying profile of ferro-surface, in Fig.3, subjected to periodic oscillation. Here the time-counting, specified at t=0, will start as the accelerated slider at the right end of stroke, x=0.012m, moves toward left and it will travel through the origin of coordinate, x=0.0m, for the duration of t=0.25s, and then be continuously decelerated to the left end of stroke, x=-0.012m, at t=0.5s. After then, the slider will return speedily and arrive at origin again at t=0.75s. Before completing the whole cycle of 1rps oscillation considered, the right -toward slider needs to be decelerated until the initial position is reached at t=1.00s. While survey from Fig.3, surface profile $\boldsymbol{\delta}$ for t=0s exhibits a right growing tendency of -0.4mm ~ 0.4mm where the minus value indicates the ferro-surface lower than the initial level. That also tells the higher surface at the right end, x=0.012m, induces a stronger hydraulic pressure to push the flow toward left, in which the right-side wall of slider will be subjected to the maximum restoring force for stretched spring. Oppositely, left lifted surface, 0.4mm ~ -0.4mm, occurring at t=0.5s provides a stronger hydraulic pressure to drive the right-toward flow as well, and here the maximum restoring force for compressed spring will be enforced. Thus these surficial distributions accessed, in agreement with the slider's dynamics discussed above, seem to be nothing surprised. In addition, the flat ferro-surface, arisen from transient static surficial kinematics induced, might be predictable as the slider, at t=0.25s and 0.75s, just travels through the origin of coordinate (x=0.0m) where a standing node of zero amplitude kept for half-wave fluctuation is developed, i.e., effective seismic-proof will be predicted here due to local surficial position is likely to be "frozen" during the periodic oscillation is in progress.

According to the results of Fig.3 and Fig.4, we will subsequently forward to analyze the vertical velocity ***v'*** distributed at ferro-surface. During the slider travels from right to left end of stroke for the duration of t=0s ~ 0.5s, the positive vertical surficial velocity on the bottom surface, located at left side of x=0, exhibits a quick- to -slow growth (t=0.0s ~ 0.25s) and then quickly drops (t=0.25s ~ 0.5s) whereas negative vertical surficial velocity on the top surface, located at right side of x=0, gives a quick- to- slow drop(t=0.0s ~ 0.25s) and then quickly rises (t=0.25s ~ 0.5s). As to the latter half of period, 0.5s ~ 1.0s, for the slider returning from left to right end of stroke, ferro-dynamics oppositely to above will be expected. Besides, the vertical stagnation delivered in Fig.4 is primarily focused on the critical profiles, t=0.0s and 0.5s of Fig.3, namely, zero rising or falling velocity occurring on the highest or lowest surface could be acknowledged as temporarily static. That could be also confirmed from the definition of Eqs.(9). On the other hand, the extremely fluctuating distributions, among all the profiles in Fig.4, are induced on the smooth flat ferro- surfaces as indicated in Fig.3 at t=0.25s and 0.75s, and which individually conveys the vertical velocity of 3mm/s ~ -3mm/s or -3mm/s ~ 3mm/s distributed within the whole length considered, and a phrase lag of one-fourth period, 0.25s, will be caused due to the existence of non- synchronization between $\boldsymbol{\delta}$ and ***v'***.

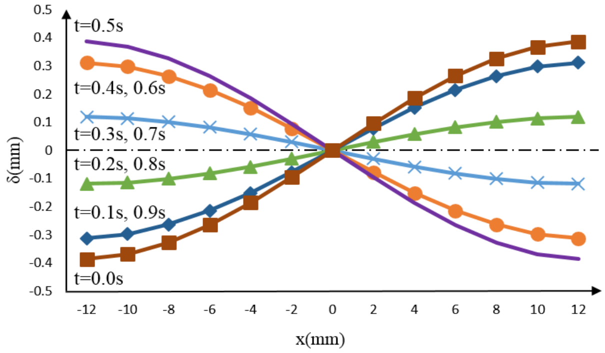

Figure 3. Periodic-surface $\boldsymbol{\delta}$ varies with duration t

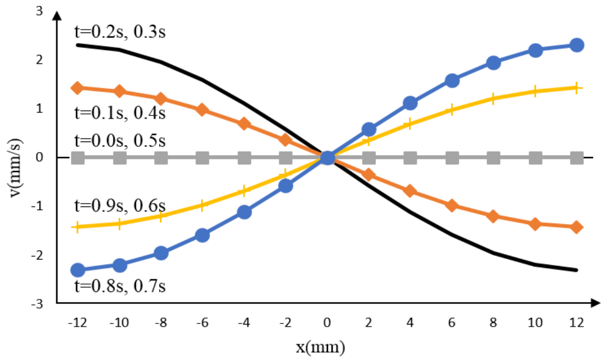

Figure 4. Vertical velocity of ferro-surface *v'* varies with duration

Finally, our discussion, based on Fig.5, will turn to quantity the relative velocity ***u'*** varying with duration t. Here the initial ***u'***, on the surface of t=0s displayed in Fig.3, shows a convex profile with the extreme magnitude of -250mm/s at x=0. That attributes to higher elevation at right surface offering a net hydraulic head to drive the flow toward left and the maximum slope is found to be distributed at x=0. While the duration of 0 ~ 0.25s is proceeded, the values of ***u'*** will be gradually resulted to be zero, as the dash lines indicated in Fig.5, due to the flat surface, without head difference shown in Fig.3, is restored at t=0.25s in which the slider just accelerates through x=0 from the left end of stroke. After then, the concaved ferro-dynamic profile, on the left- rise and right-fall surface, continues to emerge until the slider reaches at the left end of stroke at t=0.5s. During the duration of 0.25s ~ 0.5s, the left surface of higher elevation, accessed from Fig.3, provides a net hydraulic force to push the right-toward flow. i.e., the positive flow field in Fig.5 will turn out. Later, a reversed transport of relative velocity as well as surficial profile starts being induced as the latter period at t=0.5s is in progress.

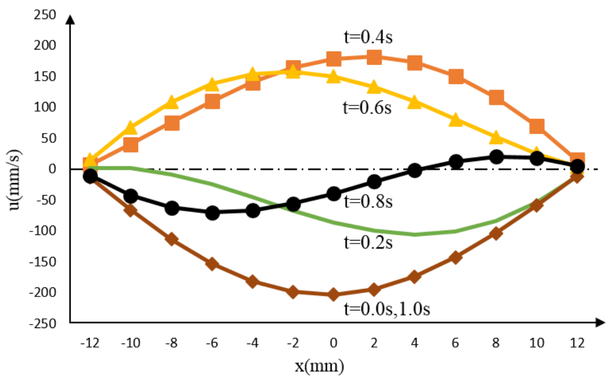

Figure 5. Relative horizontal velocity ***u'*** varies with duration t

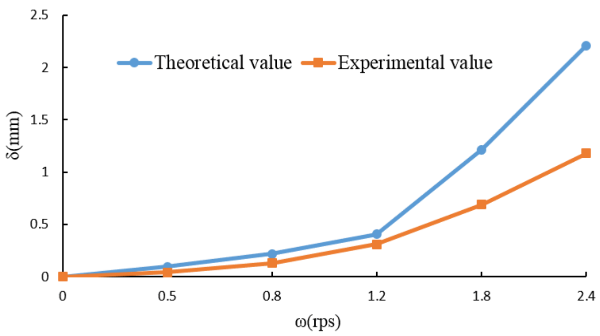

Figure 6. The comparison between theoretic and experimental surface level $\boldsymbol{\delta}$

To examine the validity of empirical model, the comparison of theoretic and experimental δ, in Fig.6, is still required before enclosing the discussion. Here a nearly linear relation held for both fluctuating amplitudes ***δ***, 0~0.38mm, are found to be consistent and their maximum relative error will be less than 10% within the oscillating frequency ***ω***, 0~ 1.2 rps, applied. However the deviation, for ***ω*** more than 1.2 rps, will be visible due to the fast growing value of analytic ***δ***, 2.5mm, much higher than experimental ***δ***, 1.3mm, appears. That might be attributed to the ferro-levitation is not so far smaller compared to the length of testing vessel that the assumption (4) regarding the ignored vertical momentum, in this study, becomes unreasonable.

4. Conclusion

Summary from above discussion, the fluctuating frequency ***ω*** is found to be a key parameter influencing on critical ferro-characteristic. Here the progressive amplitude ***δ***, vertical fluctuating velocity of surface ***v'*** is individually closely dependent on the square and cubic of ***ω*** whereas relative velocity ***u'***. vs. ***ω*** behave a linear relation. Yet these signals accessed are found to be not in synchronization, i.e., a timing lag of one-fourth period exists between signal of ***δ*** and ***v'*** or ***v'*** and ***u'***. In addition, magnetic pressure gm and kinematic viscosity ***ν***, arisen from ferro-magnetization intensified by field intensity, play a negative effect on critical ferro-response. Here the nodal position of standing wave, settled at the middle of the moving system, will be considered to hold the priority to achieve the effect of quakeproof. Besides, the ignored vertical flow momentum will make the analytic ***δ*** overestimated. Based on above findings confirmed, analytic model and experiment method, developed from this article, has provided an effective way to investigate critical characteristic of harmonic ferro-dynamics, which is also believed to be beneficial to the future research on seismic-proof.

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