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Grey-Fuzzy Regulation of a DC Motor Using Genetic Algorithms

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Abstract

A novel grey-fuzzy regulator using genetic algorithms for a dc motor is presented in this paper. The grey system theory is employed to design a grey prediction model. After the grey model predicting the feedback errors of the motor speed, a compensated energy will be generated for the fuzzy controller. Appending the grey predictor to the feedback control system, then the fuzzy inference engine receives not only the current data but also the future information. Furthermore, the optimal parameter of the grey model is acquired via genetic algorithms. Computer simulation results show that the regulation performance of the motor speed and the control energy is improved.

Keywords : Grey prediction, fuzzy regulation, genetic algorithms, dc motor.

應用基因演算法之直流馬達灰色模糊調變

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摘　要

本文提出應用基因演算法之直流馬達灰色模糊調變新方法。灰色系統理論被用於設計灰色預測模型。經由此灰色模型預測馬達轉速之回授誤差後，產生一補償電壓於模糊控制器。回授控制系統附加灰色預測器，則模糊推理引擎不只接收現在資料亦包括未來訊息。此外，經由基因演算法可獲得灰色模型之最佳參數。從電腦模擬結果證實應用基因演算法之灰色模糊控制器設計，可有效改善馬達轉速與控制能量之調變性能。

關鍵詞：灰色預測，模糊調變，基因演算法，直流馬達

I. Introduction

The industrial level and technological idea have been changed by the impact of the automation recently. An important electrical control component is the dc motor, which is a power actuator device that delivers energy to a load. Because of features such as speed controllability, well-behaved characteristics, and adaptability to various types of control methods, dc motors are extensively used in numerous servomechanisms [5]. Thus, it is very important to study the regulation of dc motors.

L.A. Zadeh first proposed the idea of fuzzy set [10]. The fuzzy inference system employing fuzzy if-then rules can control a plant using the human knowledge. Fuzzy controller has been widely used in industry for its easy realization. Much work has been done on the analysis of control rules and membership function parameters [8]. However, the traditional control strategies only adopted the previous state information as the input signal of the decision-making mechanism. This type of fuzzy control reflects the current status and is lack of adaptability.

The grey system theory was established in 1982 [2]. The grey predictor of a system can be built by only a few given data or undetermined information. Then it can be used to forecast the system outputs with high accuracy. The first-order one-variable grey model GM (1,1) is increasingly applied in many fields of engineering [6]. For example, a PD controller combined with a grey prediction model is used to control an inverted pendulum [4]. In [1], an integral variable structure controller with grey prediction is applied to compensate the speed of a synchronous reluctance motor.

In [11], a fuzzy inference system with a grey predictor is utilized to regulate the speed response of a dc motor. Using only 3 past data of speed error, the grey prediction model is built. Appending the grey predictor to the feedback loop, the fuzzy controller receives not only the current data but also the future information. This type of control reflects both the current status and the future tendency of the states. Therefore, the regulation response of the dc motor can be controlled in advance.

In this paper, a new grey-fuzzy regulation based on genetic algorithms is proposed. To enhance the accuracy of prediction, the genetic algorithms are applied to search the optimal parameter of grey model. The characteristics of genetic algorithms are the random information exchange among the population without constraint condition of the searching space [12]. Genetic algorithms employ chromosomes through three operations, reproduction, crossover, and mutations to generate offspring for next iterations. The advantages of genetic algorithms include derivative-free stochastic optimization, parallel-search procedure and applicable to both continuous and discrete problems. Thus, the performance of the grey-fuzzy regulator will be promoted after the optimal grey model is obtained.

This paper is organized as follows. In section 2, the mathematical model of a dc motor is derived. In section 3, the structure of grey-fuzzy control with genetic algorithms is constructed. In section 4, simulation results with different initial conditions are shown. Conclusion remarks are given in section 5.

II. State-Space Model of DC Motors

The dc motor converts direct current electrical energy into rotational mechanical energy. Because of high efficiency, compact size, good consumption of heat and so on, dc motors are extensively applied to many kinds of actuators [7]. The equivalent electric circuit shows that u is the armature voltage (V), R is the armature resistance (Ω), L is the armature inductance (H), e_b is the back electromotive-force voltage (V), ω is the angular velocity (rad/sec), and i is the electric current (A) of armature. The physical relation is expressed as

$$u = L\frac{di}{dt} + Ri + e_b \qquad (1)$$

$$J_m \frac{d\omega}{dt} = T_m - T_L - b_m\omega \qquad (2)$$

$$e_b = K_b\omega \qquad (3)$$

where J_m is the rotor inertia (kgf-cm-sec^2), T_m is the rotor torque (kgf-cm/A), b_m is the rotor damping ratio (g-cm/rpm), T_L is the load torque (kgf-cm/A), and K_b is the back electromotive-force constant (v・s/rad).

Substituting Eq. (3) into Eq. (1) and rearranging Eq. (2) yields

$$\frac{di}{dt} = -\frac{R}{L}i - \frac{k_b}{L}\omega + \frac{1}{L}u \qquad (4)$$

$$\frac{d\omega}{dt} = -\frac{b_m}{J_m}\omega + K_{mL} i \qquad (5)$$

where K_{mL} is the torque constant (N-m/A). Let the electric current of armature and the angular velocity be the two state variables. Then the linearized state-space model of the dc motor could be obtained:

$$\dot{X} = AX + Bu$$

$$= \begin{bmatrix} -\frac{R}{L} & -\frac{k_b}{L} \\ K_{mL} & -\frac{b_m}{J_m} \end{bmatrix} \begin{bmatrix} i \\ \omega \end{bmatrix} + \begin{bmatrix} \frac{1}{L} \\ 0 \end{bmatrix} u \qquad (6)$$

The parameters of the dc motor are listed in Table 1.

III. Grey-Fuzzy Regulation Using Genetic Algorithms

1. Fuzzy Controller

The fuzzy inference system employing fuzzy if-then rules can control a plant using the human knowledge. The design procedure of traditional fuzzy logic controller includes fuzzifier, fuzzy rule base, fuzzy inference, and defuzzifier.

In this paper, the linguistic variables are angular velocity error e_w, angular velocity w, control energy u^*, estimated error of angular velocity $\hat{e}_w$ and compensated energy Δu. The Mamdani fuzzy inference system uses minimum and maximum for T-norm and T-conorm operators, respectively. With max-min composition, the consequence $\mu_{B'}(y)$ is

$$\mu_{B'}(y) = \max_{l=1}^{m}[\mu_{A_1^l}(x_1) \wedge \mu_{A_2^l}(x_2) \wedge \mu_{B^l}(y)] \qquad (7)$$

where x_1, x_2, and y represent the fuzzy sets e_w, w, and u^*, respectively; $\mu_{A_1^l}(x_1)$, $\mu_{A_2^l}(x_2)$, and $\mu_{B^l}(y)$ mean the membership functions of e_w, w, and u^*, respectively; l is the index of membership functions for every fuzzy set; m is the number of triggered fuzzy rules.

The fuzzy inference $\mu_{D'}(z)$ for the fuzzy controller with grey prediction is

$$\mu_{D'}(z) = \max_{k=1}^{n}[\mu_{C^k}(x^*) \wedge \mu_{D^k}(z)] \qquad (8)$$

where x^* and z represent the fuzzy sets $\hat{e}_w$ and Δu, respectively; $\mu_{C^k}(x^*)$ and $\mu_{D^k}(z)$ mean the membership functions of $\hat{e}_w$ and Δu; k is the index of membership functions for every fuzzy set; n is the number of triggered fuzzy rules.

Defuzzification is used to extract a crisp value from a fuzzy set. There are a lot of methods for defuzzifying a fuzzy set. The center of gravity defuzzification is adopted in this paper [9]

$$Y^* = \frac{\sum_{i=1}^{m} \mu_{B^i}(y_i) \cdot y_i}{\sum_{i=1}^{m} \mu_{B^i}(y_i)} \qquad (9)$$

$$Z^* = \frac{\sum_{j=1}^{n} \mu_{D^j}(z_j) \cdot z_j}{\sum_{j=1}^{n} \mu_{D^j}(z_j)} \qquad (10)$$

where Y^* and Z^* represent the crisp values of control energy u^* and compensated energy Δu, respectively, i and j are the index of consequent rules.

2. Grey Predicton Model

A system is called "grey" when its part of information is unknown [3]. Grey system theory has been applied to many fields of engineering. It can be used to construct a grey model (GM) for uncertain systems with incomplete information. The GM describes a system behavior via a first order differential equation and can be served as a useful predictor with only few past data. The approach to build up the predictor of the state error is depicted as follows.

Assume that the original sequence data of the state error with 3 samples is expressed as

$$\delta x^{(0)} = \{\, \delta x^{(0)}(1), \delta x^{(0)}(2), \delta x^{(0)}(3) \,\} \qquad (11)$$

After the first order accumulated generating operation (AGO) is taken on $\delta x^{(0)}$, the sequence data $\delta x^{(1)}$ is obtained by

$$\delta x^{(1)} = \{\, \delta x^{(1)}(1), \delta x^{(1)}(2), \delta x^{(1)}(3) \,\} \qquad (12)$$

where $\delta x^{(1)}(n) = \sum_{i=1}^{n} \delta x^{(0)}(i)$, $\forall n \in \{1, 2, 3\}$.

The generating mean sequence data $Z^{(1)}$ is defined as

$$Z^{(1)}(n) = \frac{\delta x^{(1)}(n) + \delta x^{(1)}(n-1)}{2} \qquad (13)$$

$\forall$ $n \in \{2,3\}$. The grey difference equation is established as

$$\delta x^{(0)}(n) + aZ^{(1)}(n) = b \qquad (14)$$

$\forall$ $n \in \{2,3\}$. Then, the grey model of the state error is described by the first order differential equation

$$\frac{d}{dt}\delta x^{(1)}(t) + a\delta x^{(1)}(t) = b \qquad (15)$$

The parameters a and b can be estimated by least square method as

$$\begin{bmatrix} a \\ b \end{bmatrix} = (E^T E)^{-1} E^T F \qquad (16)$$

where $E=\begin{bmatrix} -Z^{(1)}(2) & 1 \\ -Z^{(1)}(3) & 1 \end{bmatrix}$, $F=\begin{bmatrix} \delta x^{(0)}(2) \\ \delta x^{(0)}(3) \end{bmatrix}$.

The solution of Eq. (15) is

$$\delta\hat{x}^{(1)}(n+1) = \left[\delta x^{(0)}(1) - \frac{b}{a}\right] e^{-an} + \frac{b}{a} \qquad (17)$$

Taking inverse accumulated generating operation (IAGO) on $\delta\hat{x}^{(1)}$, the predictive state error of the dc motor is obtained by

$$\delta\hat{x}^{(0)}(n+1) = \delta\hat{x}^{(1)}(n+1) - \delta\hat{x}^{(1)}(n)$$

$$= (1-e^{a})\left[\delta x^{(0)}(1) - \frac{b}{a}\right] e^{-an} \qquad (18)$$

3. Grey-Fuzzy Regulation

Grey prediction model is derived from the grey system theory. The main purpose of the grey model is to find the relation from a set of data and infer the tendency of the future. Therefore, the fuzzy controller uses the prediction signal of grey model as the input of inference system in this paper first. The fuzzy controller combined with a grey predictor is applied to regulate the angular velocity of a dc motor. Fig. 1 shows the block diagram of control structure.

Based on fuzzy theory, control energy $u*$ is inferred from fuzzy controller according to the input variables: angular velocity error $e_w(t)$ and angular velocity $w(t)$. The estimated error of angular velocity $\hat{e}_w(t+1)$ is obtained by sending the signal of angular velocity error $e_w(t)$ to the grey predictor. The compensated control energy Δu is inferred from another fuzzy controller according to the estimated error of angular velocity $\hat{e}_w(t+1)$. If the compensated energy Δu is added to the control energy $u*$, then the total input energy of the dc motor will be the output sum of two fuzzy controllers. Therefore the regulation response of the dc motor can be controlled in advance and the performance will be promoted.

The design algorithm of the fuzzy controller with grey prediction is illustrated as follows.

(a) Built the grey prediction model for the feedback error of the state variable $w(t)$. Measure a series of error signal: $e_w(t-2)$, $e_w(t-1)$, and $e_w(t)$. Calculate the estimated error $\hat{e}_w(t+1)$, $t \geq 3$ from equation (18).

(b) Design the fuzzy inference engine for regulation of $w(t)$. The control energy $u*(t)$ is obtained by equation (9).

(c) After the grey prediction of step (a), the estimated error $\hat{e}_w(t+1)$ is used as the input signal of another fuzzy controller. The compensated energy is derived by equation (10).

(d) The total control energy $u(t)$ of the dc motor is

$$u(t) = u*(t) + \Delta u(t+1) \qquad (19)$$

By substituting equation (19) into equation (6), the regulation response of system is acquired.

4. Genetic Algorithms

In traditional grey model, the sequence data $Z^{(1)}$ (13) is applied to derive the grey prediction model (18). However, the process of the original grey information for whiting is not optimal. To reduce the error of grey prediction, a new sequence data is introduced in this paper

$$Z^{(1)}(n) = \alpha\delta x^{(1)}(n) + (1-\alpha)\delta x^{(1)}(n-1) \qquad (20)$$

where $\alpha \in [0,1]$.

It means that the traditional grey model is just a special case of (20) for $\alpha = 0.5$. To seek the optimal value of α, the genetic algorithms (GAs) are applied. The GAs encodes each unit in a parameter space into a binary bit string called a chromosome, and each unit is connected with a “fitness” value. GAs usually keeps a set of unit as a population, and constructs a new population using genetic operators such as selection, crossover and mutation, in each generation. After getting a number of generations, the population contains members with better fitness values.

The α will be coded by a binary string and constitute a chromosome. The population is generated randomly. After solving the grey prediction equation, the grey-fuzzy controller is obtained and the fitness of the population is evaluated. In this paper, the fitness function is defined as follow:

$$PI = MIN\_offset - \sum |e| \qquad (21)$$

where PI is the fitness value, e is the error of the regulation and MIN_offset is a constant. After the fitness function is calculated, the fitness value and the number of the generation determine whether the evolution process is stopped or not.

IV. Computer Simulation

The Gaussian membership functions of fuzzy sets are employed in this paper. The number of membership functions of input e_w, w, and $\hat{e}_w$ are 3, 5, and 3, respectively. The number of membership functions of output u^* and Δu are 5 and 3, respectively. In fuzzy inference engine, Minimum and Maximum are adopted for the T-norm and T-conorm operators, respectively.

The first fuzzy controller is a two-input single-output Mamdani fuzzy model with 15 rules. These rules are expressed as

R^1: IF e_w is N and w is NB, THEN u^* is PS

R^2: IF e_w is ZO and w is NB, THEN u^* is PB

⋮

R^{14}: IF e_w is ZO and w is PB, THEN u^* is NB

R^{15}: IF e_w is P and w is PB, THEN u^* is NS

These fuzzy rules are summarized in Table 2, where NB means negative big, NS means negative small, ZO means zero, PS means positive small, and PB means positive big, respectively.

The second fuzzy controller is a single-input single-output Mamdani fuzzy model with 3 rules. These rules are expressed as

R^1: IF $\hat{e}_w$ is N, THEN Δu is P

R^2: IF $\hat{e}_w$ is ZO, THEN Δu is ZO

R^3: IF $\hat{e}_w$ is P, THEN Δu is N

These fuzzy rules are summarized in Table 3, where N means negative, ZO means zero, and P means positive, respectively.

Simulation results are obtained by Matlab software. According to two different initial conditions, the performance of regulation will be compared among the traditional fuzzy controller, the grey-fuzzy controller and the GA-based grey-fuzzy controller.

Case 1: The initial values of state variables X are $[0\ \ 1]^T$. The membership functions of linguistic variables e_w, w, and Δu are with the same universe [-2 2] as shown in Fig. 2 and Fig. 3, respectively. The membership functions of linguistic variables u^*, and $\hat{e}_w$ are with the same universe [-1 1]. First, the grey predictor estimates the error of angular velocity. According to the genetic algorithms of section 3.4, the optimal grey model is obtained. Fig. 4 shows the evolution process of regulation error function. Table 4 lists the parameters of the genetic algorithms. Fig. 5, Fig. 6 and Fig. 7 show the optimal regulation response of electric current i angular velocity w, and control energy u, respectively.

On the other hand, based on the traditional fuzzy controller and the grey-fuzzy controller, the regulation response of i, ω, and u are plotted in terms of dashed lines and dotted lines, respectively. In this computer simulation, the grey predictor launches its function after the fourth data. The average estimated error e is defined as

$$e = \frac{\sum_{4}^{N} \left| \hat{e}_w - e_w \right|}{N-3} \qquad (22)$$

where N is the number of data. Table 5 demonstrates that the GA-based grey-fuzzy controller owns the best performance than the traditional fuzzy controller and the grey-fuzzy controller.

Case 2: The initial values of state variables X are $[1\ \ 2]^T$. The membership functions of linguistic variables e_w, w, u^*, and $\hat{e}_w$ are with the same universe [-10 10]. The membership functions of linguistic variables Δu are with the universe [-12 12]. By the same design procedure of case 1, the optimal regulation response of electric current i, angular velocity w, and control energy u are plotted in terms of solid lines as shown in Fig. 8, Fig. 9 and Fig. 10, respectively.

Using the traditional fuzzy controller and the grey-fuzzy controller, the regulation response of i, ω, and u are plotted in terms of dashed lines and dotted lines, respectively. Table 5 also displays that the GA-based grey-fuzzy controller owns the best performance than the traditional fuzzy controller and the grey-fuzzy controller.

V. Conclusions

In case of system with model uncertainty, fuzzy logic is suitable for the controller design. In this paper, a grey-fuzzy controller is developed for the speed regulation of a dc motor first. The grey predictor is incorporated into a fuzzy controller. The grey model is employed to estimate the state error of motor speed such that the control signal is appropriately adjusted in advance. Furthermore, the optimal parameter of the grey model is acquired via genetic algorithms. After using the GA-based grey-fuzzy controller, either the maximum overshoot or the control energy is reduced. Computer simulations have demonstrated the effectiveness of the design

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Table 1 Parameters of the DC Motor.

$R = 2.99 \quad (\Omega)$	$L = 0.42\times10^{-3} \quad (H)$
$J_m = 1.52\times10^{-2} \quad (kg - cm^2)$	$K_b = 24.1\times10^{-3} \quad (V - s / rad)$
$K_{ml} = 5.926\times10^{-3} \quad (N - m / amp)$	$b_m = 1.18\times10^{-4} \quad (N - cm - \sec / rad)$

Table 2 Fuzzy Rule Base 1.

		e_w		
w		N	ZO	P
	NB	PS	PB	PB
	NS	ZO	PS	PB
	ZO	NS	ZO	PS
	PS	NB	NS	ZO
	PB	NB	NB	NS

Table 3 Fuzzy Rule Base 2.

		$\hat{e}_w$		
Δu		N	ZO	P
	N			N
	ZO		ZO	
	P	P		

Table 4 Parameters of Genetic Algorithms.

Population	40	Bit length	20
Generation	60	Crossover rate	0.8
Min-offset	5000	Mutation rate	0.02

Table 5 Performance Comparison of Case 1.

	Settling Time	Max. Overshoot	Max. Voltage	estimated error e
Fuzzy Controller	3.8	11.37 %	0.57	
Grey-Fuzzy Controller	3.2	7.74 %	0.18	0.24 %
GA-Grey-Fuzzy Controller	2.5	3.2 %	0.11	0.06 %

Table 6 Performance Comparison of Case 2.

	Settling Time	Max. Overshoot	Max. Voltage	estimated error e
Fuzzy Controller	4.7	17.04 %	0.48	
Grey-Fuzzy Controller	4.2	9.75 %	0.37	2.75 %
GA-Grey-Fuzzy Controller	3.1	0.01 %	0.11	0.36 %

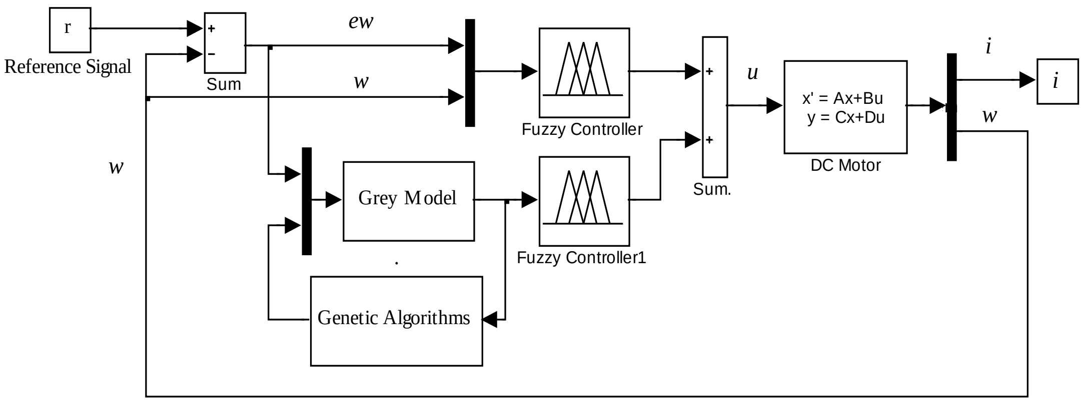

Fig. 1. The block diagram of grey-fuzzy control with genetic algorithms.

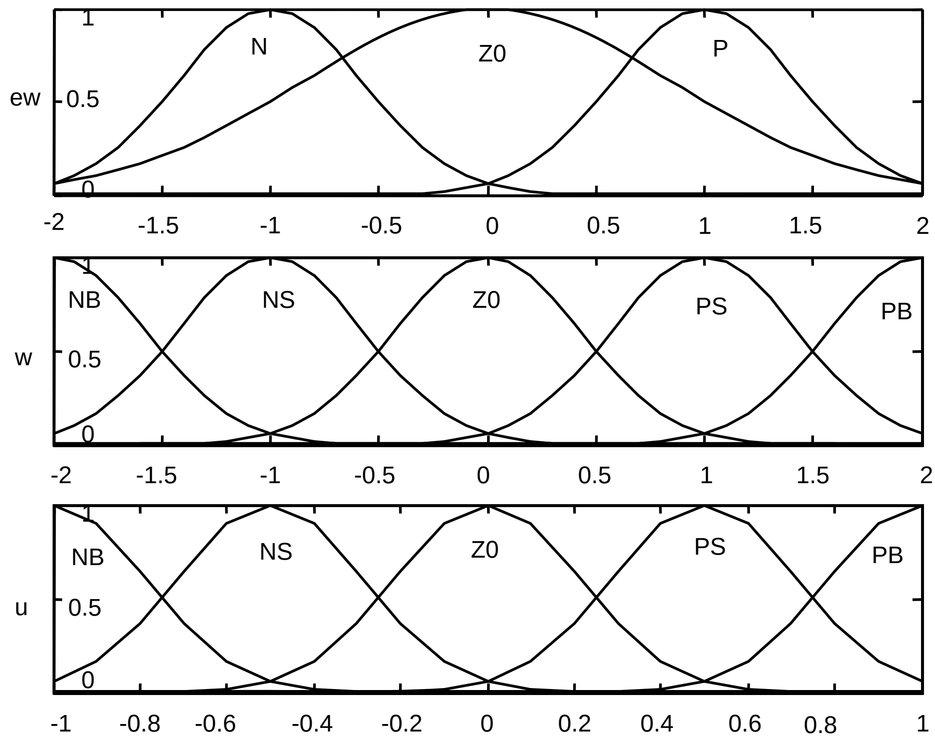

Fig. 2. Membership functions of e_w, w and $u*$.

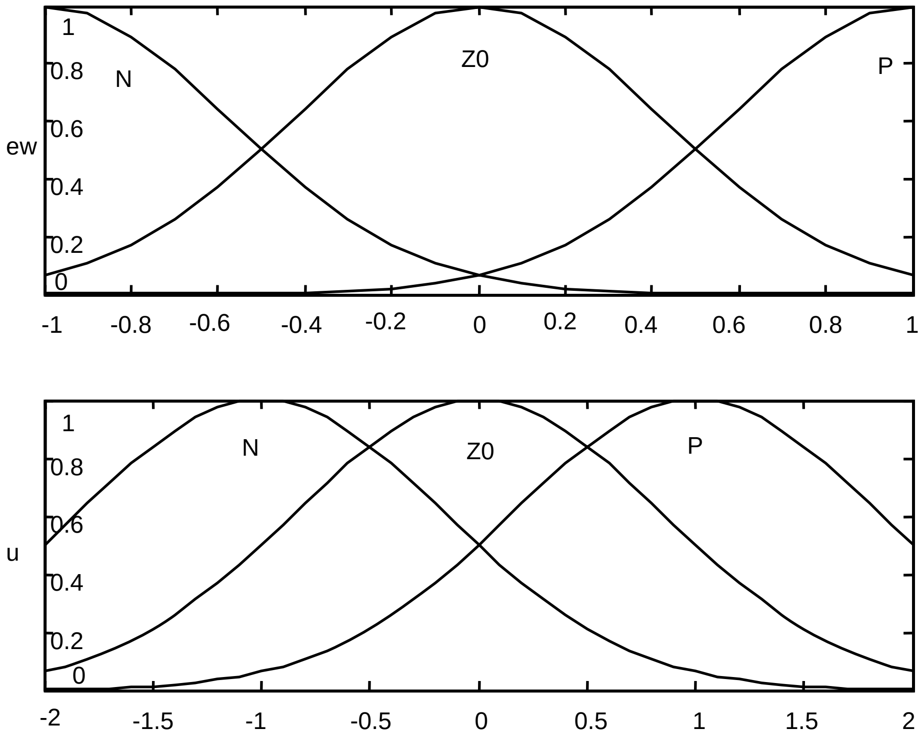

Fig. 3. Membership functions of $\hat{e}_w$ and Δu .

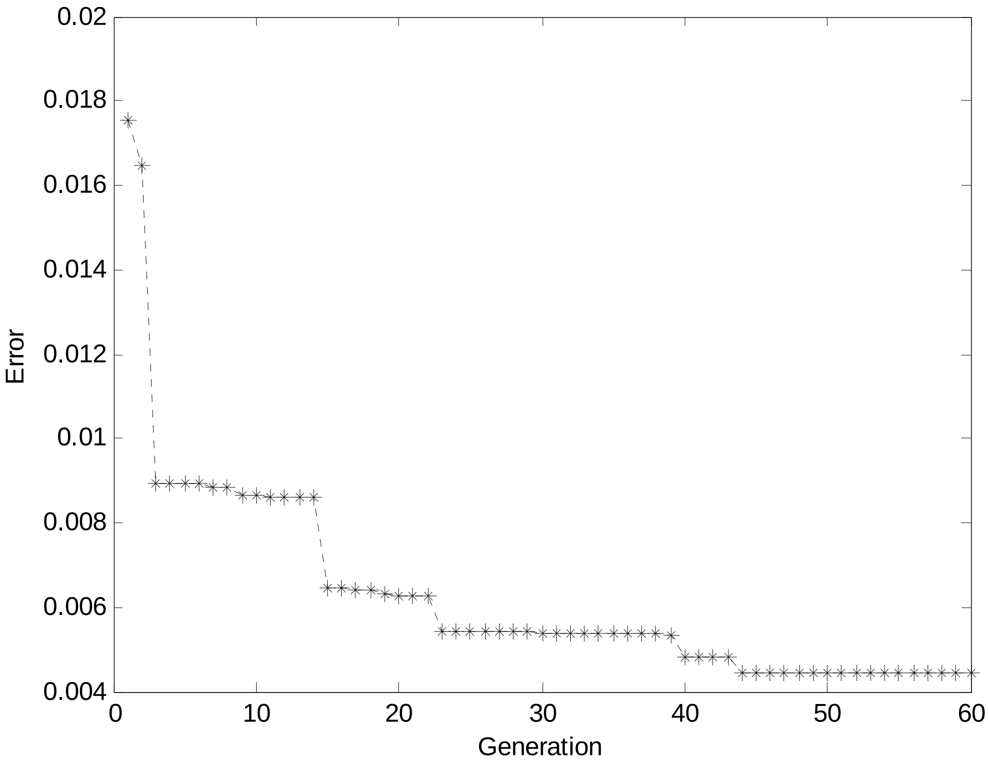

Fig. 4. Evolution process of error function.

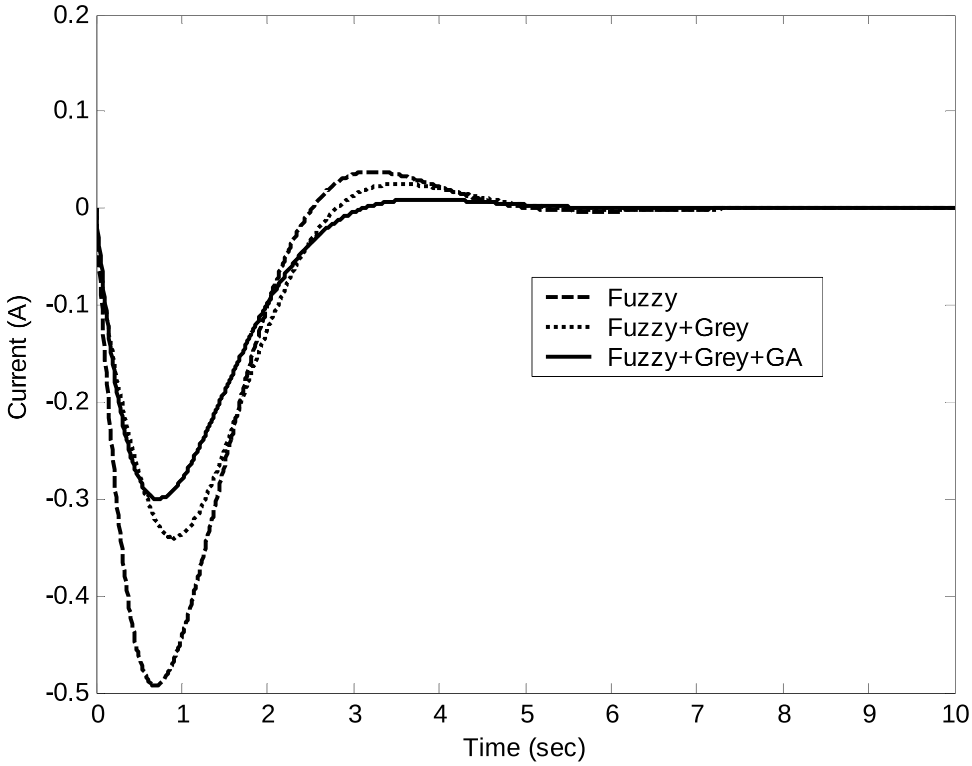

Fig. 5. Case 1: the response of electric current.

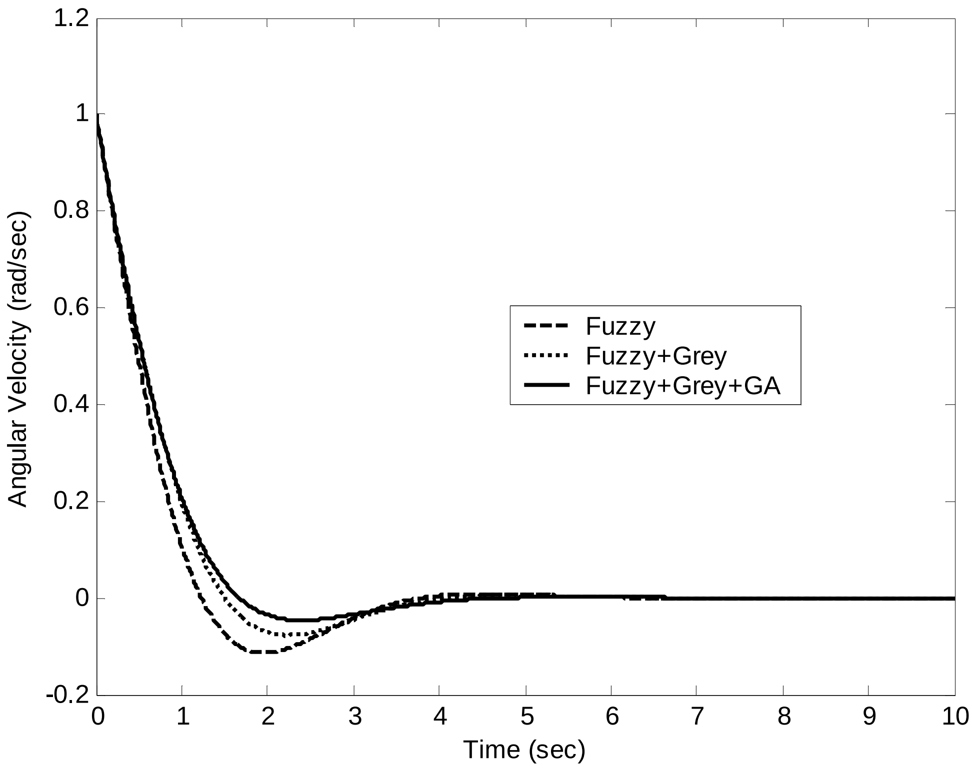

Fig. 6. Case 1: the response of angular velocity.

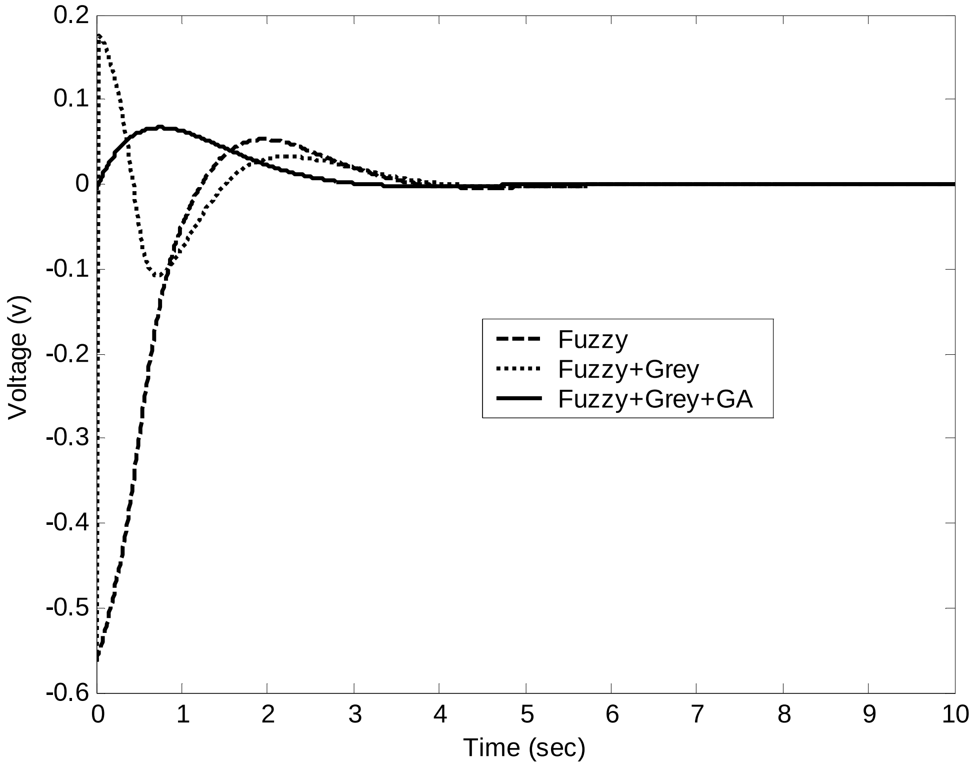

Fig. 7. Case 1: the response of control energy.

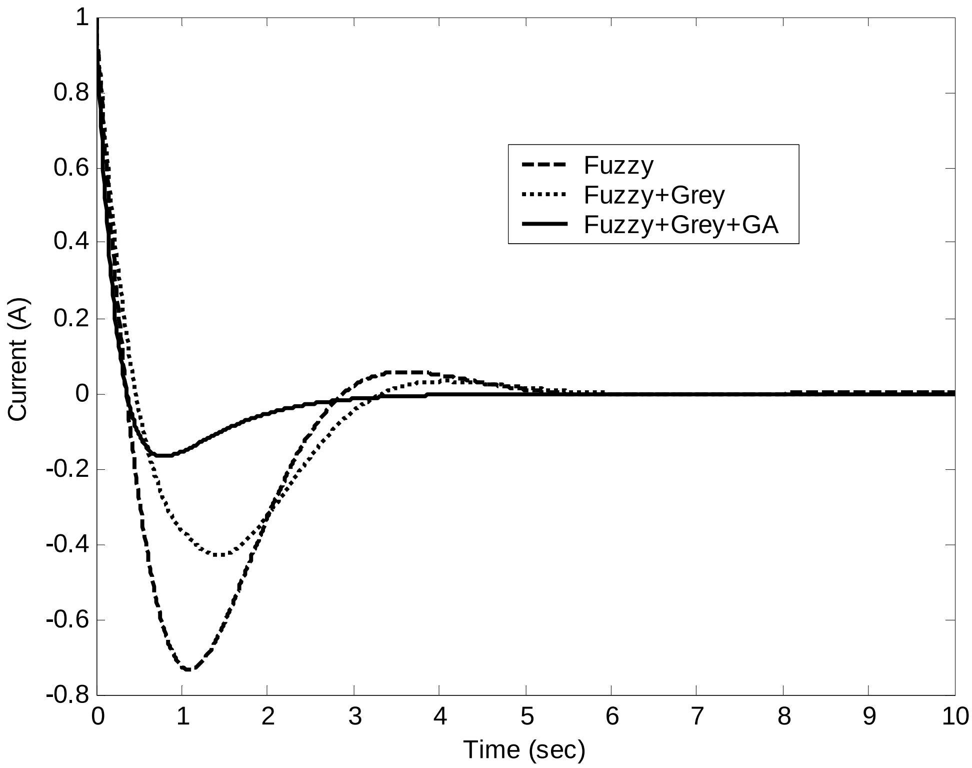

Fig. 8. Case 2: the response of electric current.

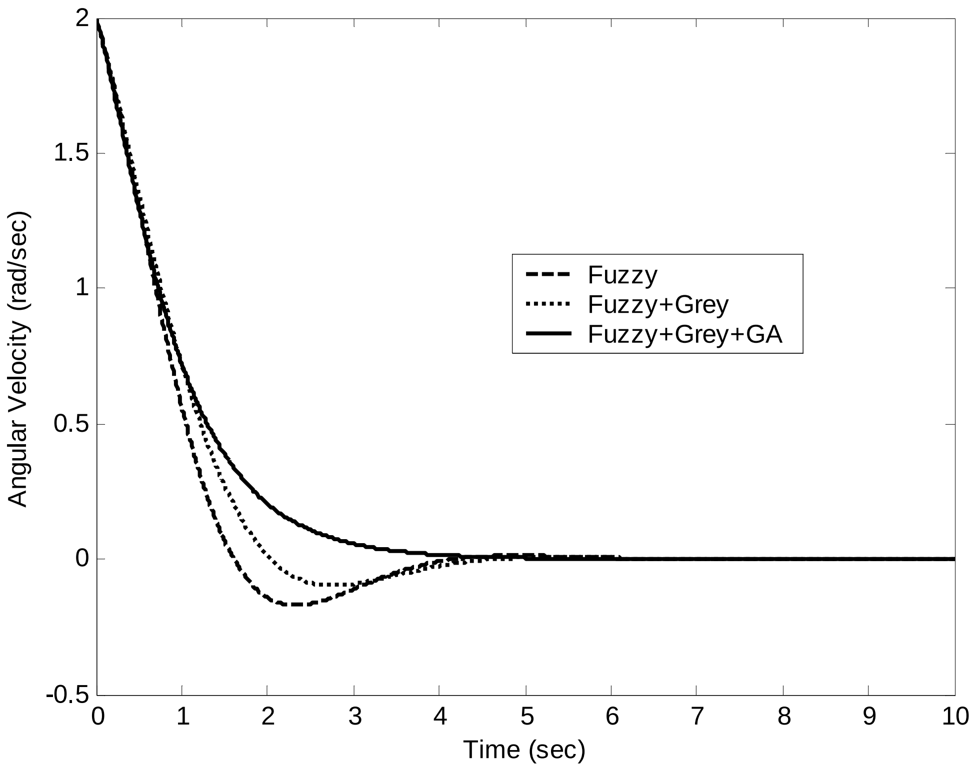

Fig. 9. Case 2: the response of angular velocity.

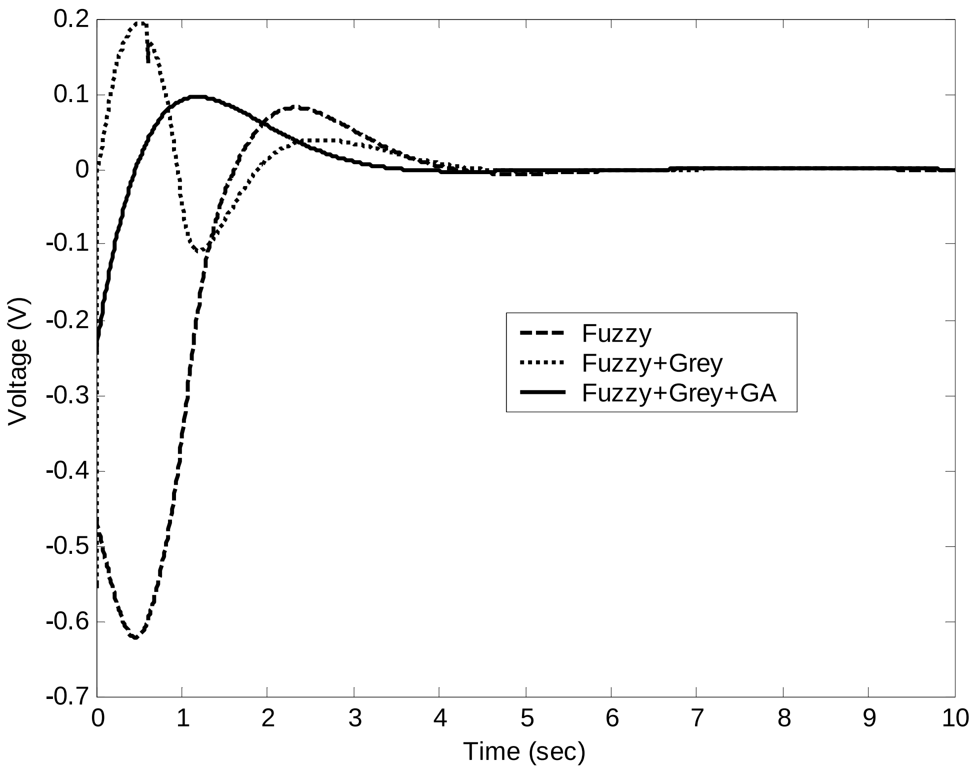

Fig. 10. Case 2: the response of control energy.