變拓樸機械系統中旋轉運動對橫向波的影響

黃國興[1]　徐偉誠[2]

1. 華夏工商專科學校機械系副教授
2. 國立宜蘭技術學院機械系副教授

摘　要

機械系統中因衝擊而產生的軸向應力波（longitudinal waves）具有相同相速度（phase velocity），吾人可利用一無窮傅力葉級數正確的描述軸向波運動。然而橫向波（transverse waves）的相速度卻具有不同的頻率，此一離散（dispersive）特性使得橫向波的分析較軸向波困難許多。本研究之主要目的在了解有限旋轉運動及運動結構的改變對機械系統的影響，並利用一含恢復係數的衝量動量方程式(impulse momentum equation)發展一套數值分析程序。其中利用動力學的虛功原理可推導出含幾何剛性的旋轉樑運動方程式，研究結果發現在低頻波中有限旋轉運動及運動結構的改變較高頻波有顯著的影響。

關鍵詞：可變拓樸結構、衝量動量方程式

The Effect of Finite Rotation on Wave Motion in Mechanical System with Topology Change

Kuo-Hsing Hwang[1] and Wei-Cheng Hsu[2]

1. Associate Professor Department of Mechanical Engineering, Hwa-Hsia College of Technology and Commerce
2. Associate Professor Department of Mechanical Engineering, National Ilan Institute of Technology

Abstract

In this investigation, a computational procedure for the analysis of impact-induced transverse waves in mechanical systems with variable kinematic structure is developed. The dispersive nature of the impact-induced transverse waves is examined and it is shown that the phase velocities of the waves have different frequencies, which depend nonlinearly on the wave number as well as the angular velocity of the beam. The effect of the mass capture is demonstrated using a simple model that consists of a rotating beam impacted transversely by a rigid mass. The equations of motion are developed using the principle of virtual work in dynamics. The jump discontinuity in the system velocity vectors as the result of impact are predicted using the generalized impulse momentum equations. The results obtained in this study indicated that the angular velocity of the beam and the change in the system topology has more significant effects on the wave velocities of low frequency waves as compared to high frequency waves.

Key Words：Mechanical System with Topology Change, Impulse Momentum Equations

I. Background and Object

Many mechanical systems are subjected to changes in the kinematic structure during their functional operations. Mass capture, mass release or sudden addition or deletion of kinematic constrains are possible sources of changes in the mechanical system topology. Examples of such systems are machine tools, weapon systems, and robotic manipulators. The purpose of this project is to develop a computational procedure for the analysis of impact-induced transverse waves in mechanical systems with variable kinematic structure.

Several investigations were concerns with the intermittent motion or the topology change of mechanical systems. Wehage and Haug [1] developed a method for dynamic analysis of systems with intermittent motion. They used impulse momentum to define jump discontinuities in system velocity for large-scale systems. Khulief and Shabana [2,3] proposed a method for dynamic analysis of large scale constrained system of mixed rigid and flexible bodies with variable structure. The equations of motion were written in the Lagrangian formulation. The displacement field of the flexible components in the system was defined using a finite set of coupled reference position and local elastic generalized coordinates. Chang and Shabana [4] studied the dynamics of spatial flexible mechanical systems with changeable topology. They developed a velocity transformation method that can be used to predict the jump discontinuity in the joint variables. Rismantab-Sany and Shabana [5] examined, both theoretically and numerically, the validity of the generalized impulse momentum approach in modeling impact or collision in the constrained motion of deformable bodies. The generalized impulse momentum equations and the kinematic constrain Jacobian matrix were used to predict the jump discontinuity in the velocity vector as well as the joint reaction forces.

The generalized impulse momentum equations were also used by Gau and Shabana [6] to examine the propagation of impact induced longitudinal waves in a constrained rotating rod. The analytical study presented by Gau and Shabana demonstrated that the large rotation of the rod causes dispersion of the longitudinal harmonic waves.

In this study, a simple model that consists of a rotating beam impacted transversely by a rigid mass is used to examine the effect of finite rotation and mass capture of mechanical systems . The simple model used in this study is shown in Figs. 1a and 1b. The configuration of the system used in this investigation is identified using two different sets of modes. The first set describes the system configuration of the system before the change in the system topology, while the second set describes the configuration of the system after the topology change. A set of interface is defined and used guarantee the continuity as the system topology changes.

II. Study Model

In this investigation the simple model shown in Figure 1 is used to examine the propagation of the impact-induced transverse elastic waves in mechanical systems with the variable kinematic structure. The system consists of a flexible beam, which is connected to the ground by a pin joint at point O, and a rigid mass which moves with constant velocity V^j before impacting the beam transversely (Figure 1a).

The beam is assumed to have length $l = 1m$, diameter $D = 0.037m$, mass density $r = 7870kg/m^3$, and the modules of elasticity $E = 2.\times10^{11} N/m^2$. The mass of the beam is denoted as m^i while that of the rigid mass is m^j. The beam is assumed to rotate with a constant angular velocity w. At time $t = 0$, the beam is subjected to a transverse impact by the mass j which remains attached with the tip point of the beam as shown in Figure 1b. In this investigation, the collision is assumed to occur first followed by the change in the system kinematic structure. Clearly, before the collision the vibration of the beam is described using a set of vibration modes which are different from the ones that are used to describe the deformation of the beam after the change in the system topology.

Partial Differential Equation The vibration equations of the rotating beam shown in Figure 1a can be solved by using the modal expansion method. If the effect of the longitudinal displacement on the transverse wave is neglected and if the angular velocity of the beam reference is assumed to be constant, the partial differential equation that governs the bending vibration is given by

$$\frac{\partial^2 v}{\partial t^2}+\frac{EI}{rA}\frac{\partial^4 v}{\partial x^4}-w^2 v=0 \tag{1}$$

where v is the transverse displacement, t is time, x is the axial coordinate. A is the cross sectional area, and I is the second moment of area. In this case, the dispersion relationship takes the form

$$b=\sqrt{jP^4-w^2} \tag{2}$$

where â is the frequency, P is the wave number, and $j=\frac{EI}{rA}$. If the angular velocity is equal to zero, the preceding equation reduce to

$$b=\sqrt{j}\,P^2 \tag{3}$$

which is the dispersion relationship obtained in the case of nonrotating beams [7]. Figure 2 shows the dispersion relationship of Eq.2. It is clear from the results presented in this figure that as the angular velocity of the beam reference increases, the frequency â decreases.

Using the method of separation variables, the transverse displacement v can be written

$$v(x,t)=\sum_{k=1}^{n} f_k(x)[q_f(t)]_k \tag{4}$$

where $v(x,t)$ is the transverse displacement, f_k is the kth space-dependent eigenfunction, $[q_f(t)]_k$ is the kth time dependent modal coordinate, and n is the number of the vibration modes. Before the change in the system topology, the boundary conditions of the clamped-free beam are used and given by

$$v(0,t)=\frac{\partial v(0,t)}{\partial x}=0 \tag{5}$$

$$\frac{\partial^2 v(l,t)}{\partial x^2}=\frac{\partial^3 v(l,t)}{\partial x^3}=0 \tag{6}$$

Using these boundary conditions, the eigenfunction $f_k(x)$ is given by

$$f_k(x)=\cos P_k x-\cosh P_k x-s_k(\sin P_k x-\sinh P_k x) \tag{7}$$

The parameter P_k and s_k can be numerically computed using the following formulas as

$$\cos P_k l\cosh P_k l+1=0 \tag{8}$$

$$s_k=\frac{\cos P_k l+\cosh P_k l}{\sin P_k l+\sinh P_k l} \tag{9}$$

Substituting Eq.4 into Eq.1, and using the cantilever mode shapes of Eq.7, a finite dimensional model for the rotating beam can be developed. The dynamic differential equations of the rotating beam can be written as

$$M_k(\ddot{q}_f)_k+(K_k-w^2M_k)(q_f)_k=0 \tag{10}$$

where M_k and K_k are, respectively, the mass and stiffness model coefficients for the Kth mode which can be evaluated using the mode shape of Eq.7.

III. Impact-induced Waves

If the geometry of the impacting surfaces and friction between the two impacting bodies are not considered, the generalized impulse and the jump discontinuities in the system variables can be predicted using the algebraic generalized impulse momentum equations. Furthermore, impact between the rigid mass and the rotating deformable beam is assumed to occur in a very short interval such that the system configuration does not appreciably change. In this case the initial conditions for the Eq.10 as the result of the transverse impact are defined as

$$[q_f(0)]_k = 0 \tag{11}$$

$$[\dot{q}_f(0)]_k = [\Delta \dot{q}_f]_k \tag{12}$$

where $\Delta \dot{q}_f$ is the jump discontinuity in the velocity defined by Palas, Hsu, and Shabana [8].

The accuracy of the jump discontinuity in the velocities depends on the number of vibration modes. Figure 3 shows the jump discontinuity in the velocity along the beam length as a function of the dimensionless coordinate $x = x/l$ for different number of modes.

IV. Change in System Topology

After the change in the system kinematic structure, new sets of vibration modes must be used to describe the transverse deformation of the beam with a mass attached to its end. Therefore, after the topology change, the transverse deformation of the beam can be written as

$$\bar{v}(x,t) = \sum_{k=1}^{n} \bar{f}_k(x)[\bar{q}_f(t)]_k \tag{13}$$

where (¯) refers to the system parameters after the change in the kinematic structure and the eigenfunctions $\bar{f}_k(x)$ are determined using the following boundary conditions

$$\bar{v}(0,t) = 0 \tag{14}$$

$$\frac{\partial \bar{v}(0,t)}{\partial x} = 0 \tag{15}$$

$$\frac{\partial^2 \bar{v}(l,t)}{\partial x^2} = 0 \tag{16}$$

$$EI\frac{\partial^3 \bar{v}(l,t)}{\partial x^3} = m^j \frac{\partial^2 \bar{v}(l,t)}{\partial t^2} \tag{17}$$

Using Eq.13 and applying the boundary conditions of Eq.14~17, lead to the definition of the eigenfunction $\bar{f}_k(x)$ as

$$\bar{f}_k(x) = \cos \bar{P}_k x - \cosh \bar{P}_k x - \bar{s}_k(\sin \bar{P}_k x - \sinh \bar{P}_k x) \tag{18}$$

where $\bar{P}_k$ is determined using the frequency equation

$$f(\bar{a}_k) = \cos \bar{a}_k \cosh \bar{a}_k + \bar{a}_k g \cos \bar{a}_k \sinh \bar{a}_k - \bar{a}_k g \cosh \bar{a}_k \sin \bar{a}_k + 1 = 0 \tag{19}$$

in which $\bar{a}_k = \bar{P}_k l$ and $\bar{s}_k$ is calculated using the following formula

$$\bar{s}_k = \frac{\cos \bar{P}_k l + \cosh \bar{P}_k l}{\sin \bar{P}_k l + \sinh \bar{P}_k l} \tag{20}$$

where $g = m^j / m^i$ is the mass ratio. The mode shapes of the system after the change in the system kinematic structure must satisfy the following orthogonality conditions

$$\int_0^l rA\bar{f}_k\bar{f}_r dx + m^j \bar{f}_k(l)\bar{f}_r(l) = \begin{cases} 0 & k \neq r \\ \bar{M}_k & k = r \end{cases} \tag{21}$$

where $\bar{M}_k$ is the modal mass coefficient.

Interface Condition In order to guarantee a smooth transition from one configuration space to another, a set of compatibility or interface conditions must be applied. This set of conditions can be used to define the jump discontinuities in the system variables associated with the new configuration. The velocity of the beam after change in topology can be written as

$$\dot{\bar{v}}(x,0)=\sum_{k=1}^{n}\bar{f}_k(x)[\dot{\bar{q}}_f(0)]_k \tag{22}$$

where $[\dot{\bar{q}}_f(0)]_k$ is the modal velocities after the topology change. Using the orthogonality conditions and Eq.22, the modal velocities can be written in terms of the velocity $\dot{\bar{v}}(x,0)$ as

$$[\dot{\bar{q}}_f(0)]_k=\frac{1}{\bar{M}_k}[\int_0^l rA\bar{f}_k(x)\dot{\bar{v}}(x,0)dx+m^j\bar{f}_k(l)\dot{\bar{v}}(l,0)] \tag{23}$$

It is assumed that the change in the system topology occurs immediately after impact. Unlike impact, the change in topology is not accompanied by jump in the system velocities. Therefore, the velocities in the beam before and after the change in the system topology remain unchanged. That is

$$\dot{\bar{v}}(x,0)=\dot{v}(x,0)=\sum_{k=1}^{n}f_k(x)[\dot{q}_f(0)]_k \tag{24}$$

Substituting Eq.24 into Eq.23, the jump in the modal variables associated with the new configuration can be defined as

$$[\bar{q}_f(0)]_k=0 \tag{25}$$

$$[\dot{\bar{q}}_f(0)]_k=\frac{1}{\bar{M}_k}\{\int_0^l rA\bar{f}_k(x)[\sum_{i=1}^{m}f_i(x)(\Delta\dot{q}_f)_i]dx+m^j\bar{f}_k(l)[\sum_{i=1}^{m}f_i(l)(\Delta\dot{q}_f)_i]\} \tag{26}$$

V. Transverse Wave Motion

The differential equations of motion expressed in terms of the new modal coordinates of the system are

$$\bar{M}_k(\ddot{\bar{q}}_f)_k+(\bar{K}_k-w^2\bar{M}_k)(\bar{q}_f)_k=0 \tag{27}$$

where $\bar{K}_k, k=1,2,\ldots,n$ are the stiffness modal coefficients and are defined as

$$\bar{K}_k=\int_0^l EI\bar{f}_k''^2(x)dx \tag{28}$$

By using the initial conditions of Eq.25 and 26, Equation 27 can be solved for the elastic coordinates as

$$[\bar{q}_f(t)]_k=\bar{D}_k\sin\bar{b}_k t \tag{29}$$

where

$$\bar{D}_k=\frac{[\dot{\bar{q}}_f(0)]_k}{\bar{b}_k},\quad \bar{b}_k=h\bar{P}_k^2\sqrt{1-\bar{h}_k^2} \tag{30}$$

in which the dimensionless rotation wave number $\bar{h}_k$ and the parameter h are defined as

$$\bar{h}_k=\frac{w}{h\bar{P}_k^2},\quad h=\sqrt{\frac{EI}{rA}}=\sqrt{j} \tag{31}$$

It is clear from Eq.30. If the angular velocity of the beam becomes large, it will cause negative stiffness coefficients which lead to exponentially increasing unstable solution. This occurs for mode k if $w > h\bar{P}_k^2$.

The phase velocity of the kth mode can be expressed in terms of the wave number as

$$(C_p)_k = \frac{\bar{b}_k}{\bar{P}_k} = h\bar{P}_k\sqrt{1-\bar{h}_k^2} \tag{32}$$

Equation 32 shows that if w , or equivalently $\bar{h}_k$, is equal to zero, $(C_p)_k$ is equal to the phase velocity in the case of nonrotating beams. Figure 4 displays the phase velocity as a function of the wave number for different values of the angular velocity w . It is clear from the results presented in this figure that the phase velocities of the waves decreases as the angular velocity of the beam increases. It is clear from Eq.32 that the phase velocities depend only on the material and the dimension of the beam as well as the finite rotation and the wave number.

After the change in the system topology, the group velocity of the kth mode can be written as

$$(C_g)_k = \frac{2h\bar{P}_k}{\sqrt{1-\bar{h}_k^2}} \tag{33}$$

Figure 5 shows the group velocity as a function of the wave number for different values of the angular velocity w .

VI. Summary and Conclusion

In this investigation, a computational procedure for the analysis of impact-induced transverse waves in mechanical systems with variable kinematic structure is developed. A simple mechanical system that consists of a rotating beam impacted transversely by a rigid mass is used to examine the effect of finite rotation and mass capture of mechanical systems . The effects of the shear deformation and rotary inertia are neglected. The equations of motion of the rotating elastic beam are developed using the principle of virtual work in dynamics. The jump discontinuity in the system velocities as the result of impact is predicted using the generalized impulse momentum equations. The configuration of the system used in this investigation is identified using two different sets of modes. The first set describes the system configuration of the system before the change in the system topology, while the second set describes the configuration of the system after the topology change. A set of interface is defined and used guarantee the continuity as the system topology changes. These conditions are used to define the jump in the new set of modal velocities as the result of impact. The dispersive nature of the impact-induced transverse waves is examined and it is shown that the phase velocities of the waves have different frequencies, which depend nonlinearly on the wave number as well as the angular velocity of the beam. It was found that the change in the system topology has more significant effects on the wave velocities of low frequency waves as compared to high frequency waves. The analysis presented in this investigation also demonstrated that the change in the wave velocities as the result of the change in the system topology is more significant in rotating beams as compared to nonrotating beams.

VII. Acknowledgement

The study was supported by the National Science Council of Taiwan, the Republic of China through the grant number NSC 88-2212-E-197-002.

VIII. Reference

1. Wehage, R. C. and Haug, E. J. (1982), "Dynamic Analysis of Mechanical Systems with Intermittent Motion," ASME Journal of Mechanical Design, Vol. 104, pp.778-784.
2. Khulief, Y. A. and A. A. Shabana (1986), "Dynamic Analysis of Constrained System of Rigid and Flexible Bodies with Intermittent Motion," ASME Journal of Mechanism, Transmissions, and Automation in Design, Vol. 108, pp. 38-45.

3. Khulief, Y. A. and A. A. Shabana (1986), “Dynamic Analysis of Multibody Systems with Variable Kinematic Structure,” ASME Journal of Mechanism, Transmissions, and Automation in Design, Vol. 108, pp. 167-175.
4. Chang, C. W. and Shabana, A. A. (1990), “Spatial Dynamics of Deformable Multibody Systems with Variable Kinematic Structure,” parts 1 and 2, ASME Journal of Mechanical Design, Vol. 112, pp. 153-167.
5. Rismantab-Sany, J. and Shabana, A.A. (1990),“On the Use of the Momentum Balance in the Impact Analysis of Constrained Elastic System,” Journal of Sound and Vibration, Vol. 112, pp. 119-126.
6. Gau, W. H. and Shabana, A. A. (1992), “Effect of Finite Rotation on the Propagation of Elastic Waves in Constrained Mechanical Systems,” ASME Journal of Mechanical Design, Vol. 114, pp. 384-393.
7. Whitham, G. B. (1974), Linear and Nonlinear Waves, Wiley, New York, U.S.A.
8. Palas, H., Hsu, W.C., Shabana, A.A. (1992), “On the Use Momentum Balance and Assumed Modes Method in Transverse Impact Problems,” *ASME Journal of Vibration and Acoustics*, Vol. 114, pp. 364-372.
9. Hsu, W. C. and Shabana, A. A. (1993), “Finite Element Analysis of Impact Induced Waves in Rotating Beams,” Journal of Sound and Vibration, Vol. 168, pp. 335-369.
10. Huang, R. C., Haug, E. J., and Andrew, J. G. (1978), “Sensitivity Analysis and Optimal Design of a Mechanical System with Intermittent,” *ASME Journal of Mechanical Design*, Vol. 100, pp. 492-499.
11. Hunt, K. H., and Crossley, F. R. E. (1975), “Coefficient of Restitution Interpreted as Damping in Vibroimpact,” *ASME Journal of Applied Mechanics*, Vol. 53, pp. 1-4.
12. Kukreti, Feng, C. (1984), “Dynamic Response Analysis of Linear Structural Systems Subject to Component Changes,” *Computers and Structures*, Vol. 18, pp. 963-876.
13. Shabana, A. A., Gau, W. H. (1993), “Propagation of Impact-Induced Longitudinal Waves in Mechanical Systems With Variable Kinematic Structure,” *ASME Journal of Vibration and Acoustics*, Vol. 115, pp1-8.
14. Shabana, A. A., Wehage, R. (1983), “A Coordinate Reduction Technique for Dynamic Analysis of Spatial Structures with Large Angular Rotations,” *Journal of Structural Mechanics*, pp.401-431.
15. Yigit, A. S., Ulsoy, A. G., Scott, R. A. (1990), “Dynamics of Radially Rotating Beam with Impact, Part 1 and 2,” *ASME Journal of Vibration and Acoustics*, Vol. 112, pp. 65-77.

91 年 09 月 17 日投稿
91 年 10 月 16 日接受

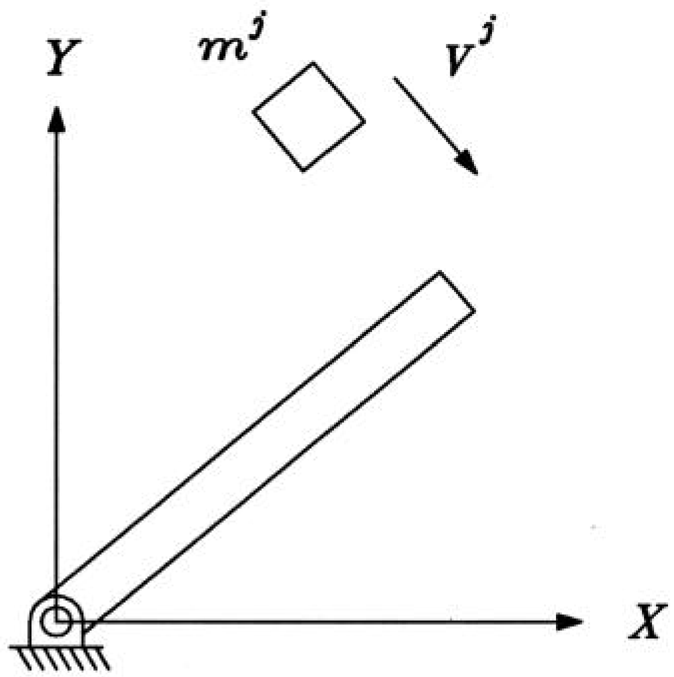

Fig 1a　Rotating beam model before impact

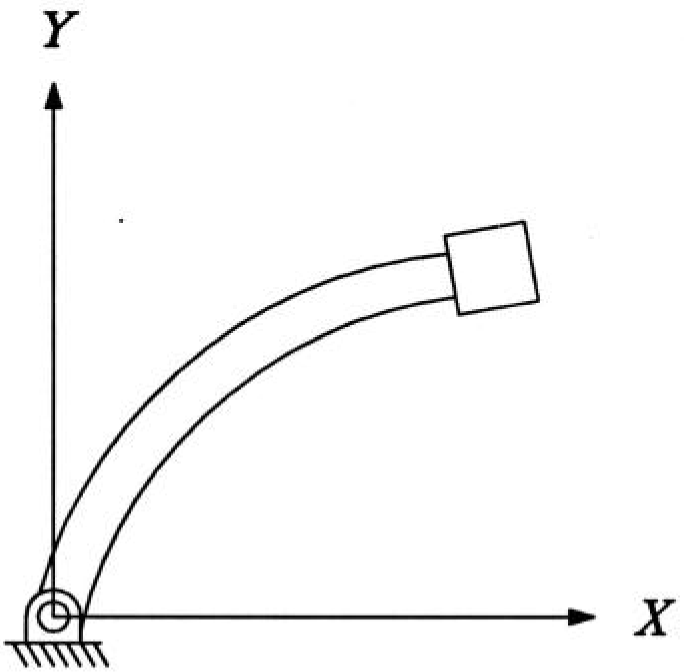

Fig 1b　Rotating beam model after impact

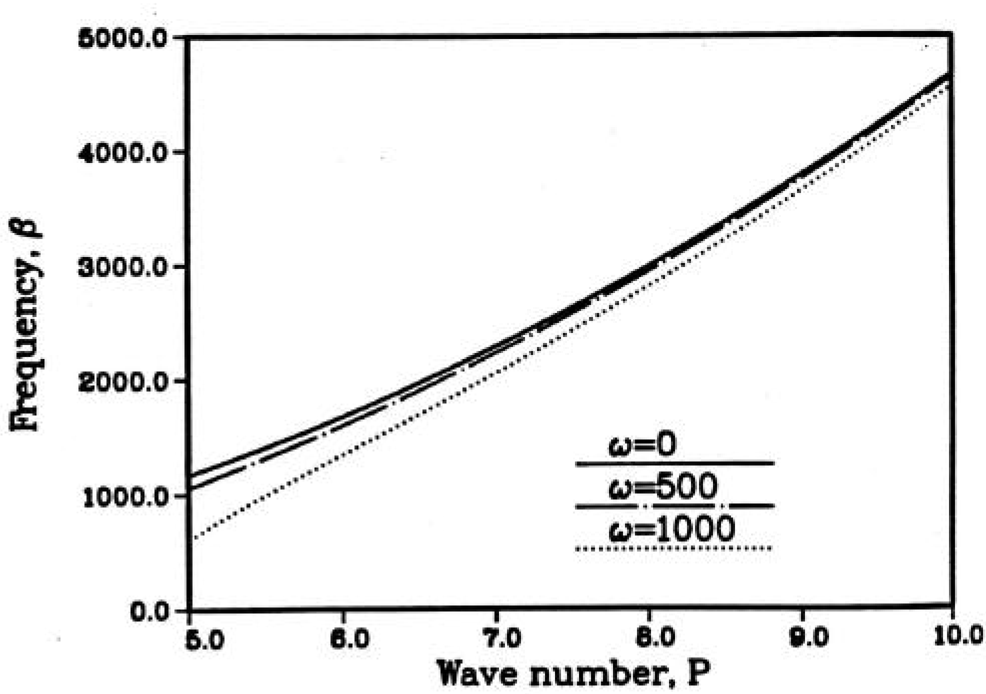

Fig 2　Effect of the angular velocity w on the dispersive relation

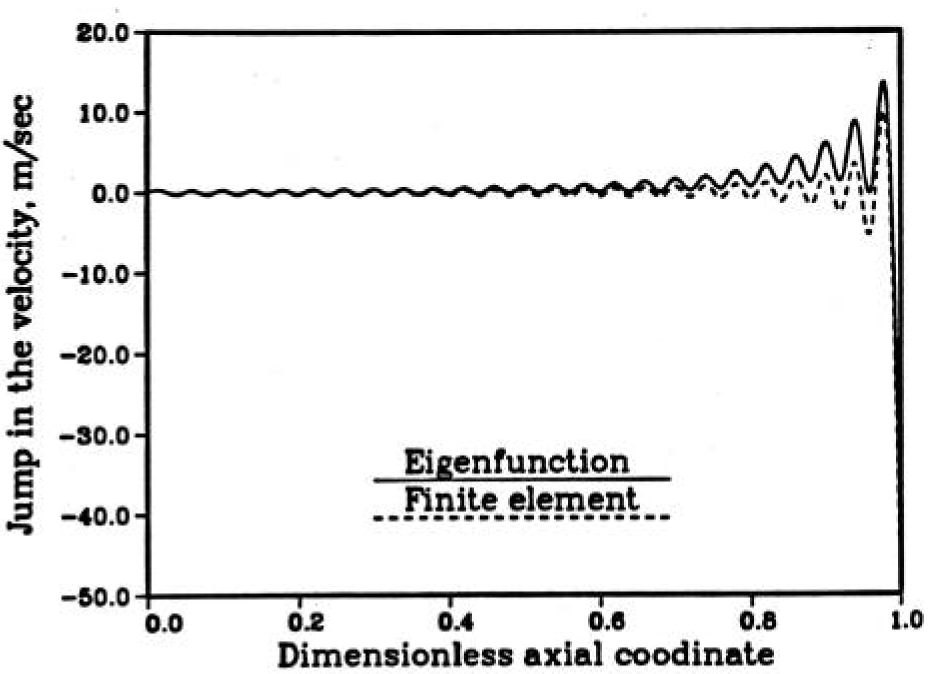

Fig 3　Jump in the velocity ($e = 0, g = 0.2$)

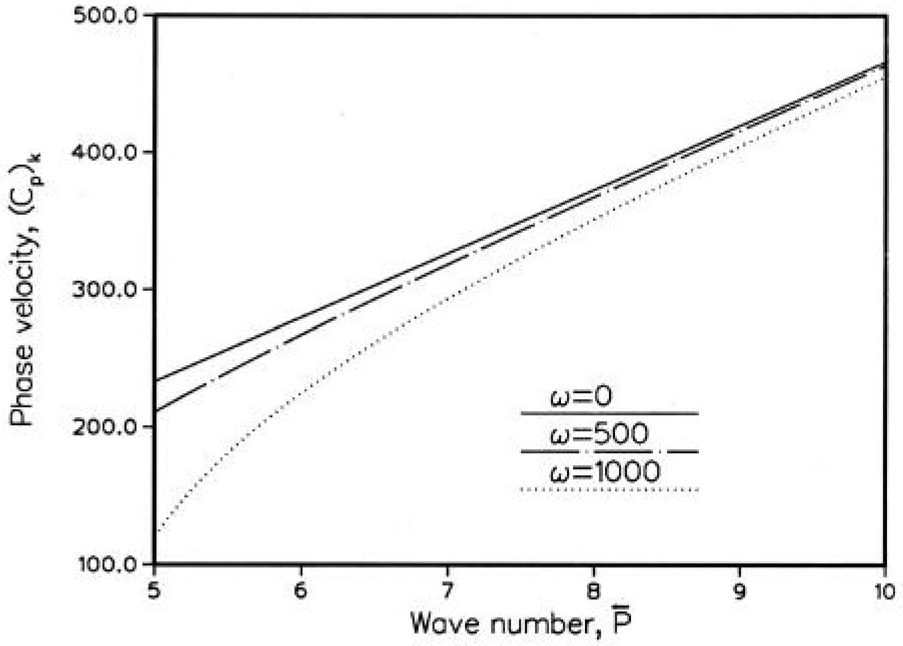

Fig 4 Effect of the angular velocity w on the phase velocity C_p

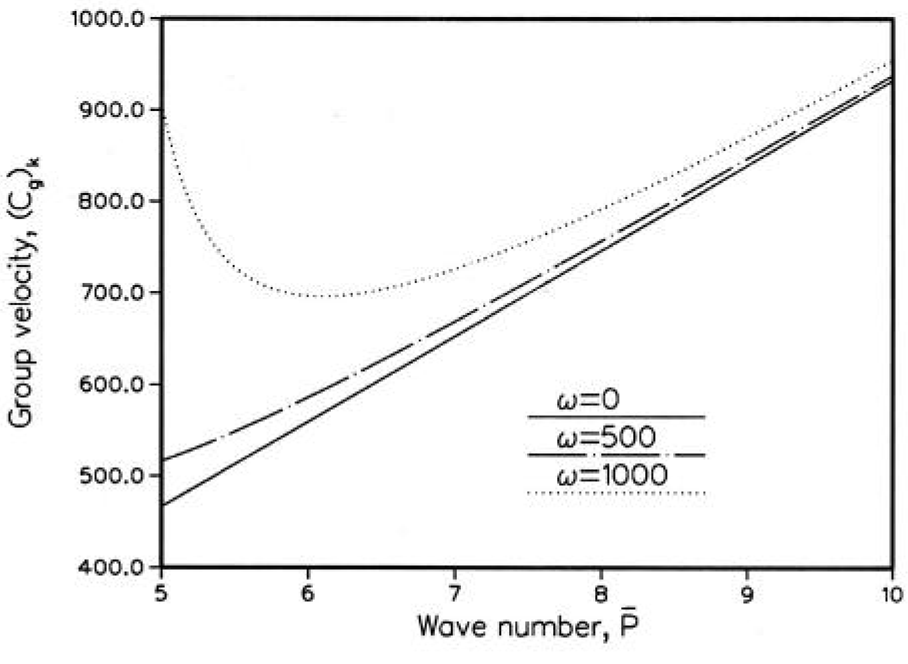

Fig 5 Effect of the angular velocity w on the group velocity C_g

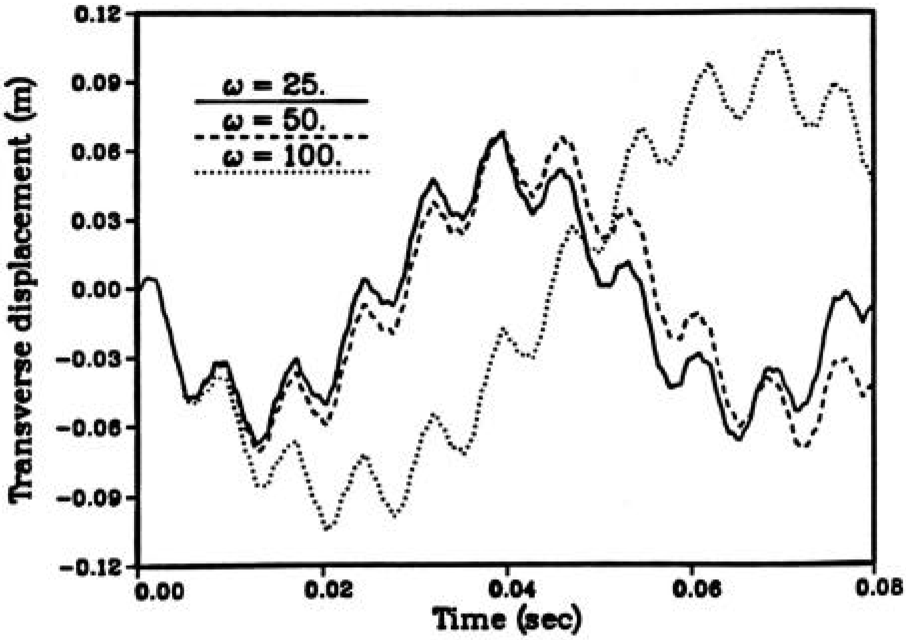

Fig 6　Transverse displacement of the mid-point $x = 0.5$ for different angular velocities w ($g = 0.2$)

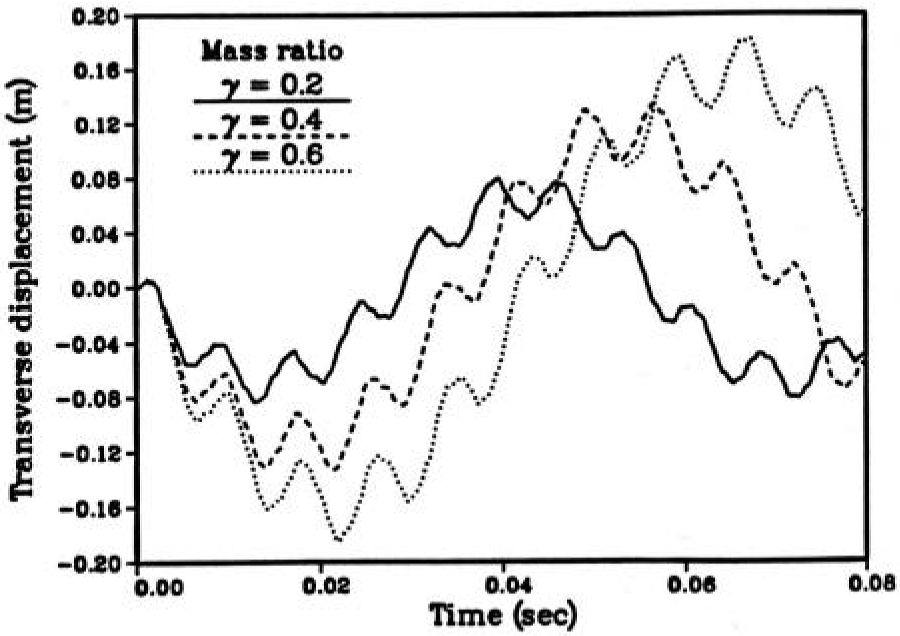

Fig 7　Transverse displacement of the mid-point $x = 0.5$ for different mass ratio g ($w = 50$)