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Correct the Venturi – Meter Equation Using a Differential Volume Approach

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ABSTRACT

A differential varied piping- volume approach, in this study, is developed to calibrate the measurement of venture manometer. Unlike the traditional venturi – meter formula on Bernoulli's equation as well as mass conservation used to predict piping flow rate, the significant deviation of estimated head loss, arisen from the flow in contracted region or divergent portion of venture tube, is usually experienced and which could be effectively improved by present model proposed with the variable of pipeline diameter has been taken into account seriously. Consequently, the relative errors, compared to experimental result, has been further reduced from 50 % to 25 % during the working conditions of inlet flow velocity with 5.2m/s ~12.2 m/s. In other words, both the vortex and turbulent jet evolve from the throat of venturi tube might be well corrected in present formula developed and which also avoids unreasonable outcomes accessed from the measurement used by hot wire meter or overestimation from previous empirical model with uncertain discharging coefficients.

Keyword: Bernoulli's equation,　turbulent jet,throat

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1. INTRODUCTION

Due to the special advantage of energy transform without additional energy loss, venture tube has been taken as an effective flow-rate meter and widely applied to hydraulic field since before. In 1996, the first limitation on Bernoulli equation applicable to steady flow was made [1]. Here the flow induced during transient start-up or shut-down period is taken as unsteady motion and related discussion was excluded and left to be conferred. To correct the accessibility of Bernoulli equation in fluid application, a modify model with discharging coefficient was determined experimentally in [2]. By this way, an analytic solution might be well approached after a tedious solving procedure goes through, that is, the coefficients predicted from the empirical polynomial series in power law should be determined in advance. Another hypothetical analysis on cavitating flows in the Venturi tube, as a theoretic extension to multi-phase problem, was proposed by [3]. In which, an extra energy dissipation will be expected as the rapid formation and collapse of vapor pockets occurs at the pipe throat where the maximum negative pressure appears. Thus the potential solution accessed from traditional model seems to be inadequate to handle the phase change problem except that a full differentiated equation governing the variation of pipe geometry might be reformulated. Kang etc.[4] initiated an virtual study of venturi device where Picot-probe together with Bernoulli equation ,responsible for the measurement of local flow velocity, is used to estimate the difference of liquid elevation in both columns of U tube and then relevant loss coefficients for various components as well as hydraulics grade line, stating the interchange of flow mechanical energy, could be also graphically illustrated. Alternatively, venture-meter embedded with other mechanical parts also possesses some special utility for environmental, mechanical and chemical purpose [5-7]. For example, it might be as a flow rate meter used in the carburetor of Otto or Diesel engine, a low-pressure inducer of vacuum cleaner to remove the air pollutant, an refrigerant regulator in cooling system or a magnetic nozzle of free jet in printer etc. Recall from previous study, an empirical formula, developed by traditional Bernoulli equation, associated with flow continuity is popularly utilized. Unfortunately, such potential model mentioned above easily leads to an overestimated result due to the ignorance of geometric parameter in energy balance. To improve above performance, flow in the region of pipe contraction and divergent portion, in this article, will be rearranged into a full differentiated form. By means of the modified transformation, an exact solution of flow rate (flow velocity) as well as head loss estimated might be well corrected.

2. ANALYSIS

Prior to embark on theoretic analysis and outline experimental procedure, several reasonable assumptions, without losing overall characteristic, should be made beforehand.

2.1 Assumptions

1. To prevent the evolution of discharge vortex, Reynold number of piping flow, less than 10000, should be limited, i.e., the fluid could be dealt with an incompressible flow if Mach number less than 0.1 is considered in this study.
2. After the region of flow-entrance length, piping flow, deliveries a parabolic profile instead of uniform distribution, will achieve a steady state.
3. Just for the accuracy in use, venturi meter preceded by the pipeline system of length about 30 times as the value of diameter is installed. Thus the formation of vortex and divergent turbulence might be further restricted.
4. Owing to the axial characteristic length is much greater than scales in other directional, only the momentum analysis along streamline direction will be considered in this model.

2.2 Governing equations

At the beginning, let's proceed to constitute the theoretic model in which the steady flow element, as display in Fig.1, is designated as a finite stream tube with variable cross-section area.

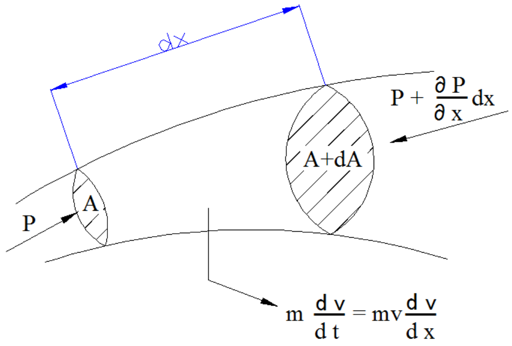

Fig. 1 Schematic of flow element

Refers to the scalar analysis made in assumption of (4), our concern will be focused on steady flow along the streamline, and the force acting on a finite stream tube in the direction of streamlines is found to be a driving force that tends to accelerate fluid mass. Apply Newton's second law *F=ma*, we will get the equivalence governing hydrodynamics problem in Eqs.(1) and a simpler expression in Eqs.(2), after further arrangement, could be formulated consequently. That tells the induced acceleration primarily depends on pressure forces on the both ends of element. Here symbol *p* indicates hydraulic pressure , *u* means the instant flow velocity, *A* is then regarded as the cross-section area varing with the traveling distance and *dx* represents the length of finite stream tube.

$$PA - \left(P + \frac{\partial P}{\partial X}dx\right)(A + dA) = \rho\left(A + \frac{1}{2}dA\right)dx\left(u\frac{du}{dx}\right) \tag{1}$$

$$-dPA = \rho A d\frac{1}{2}u^2 \tag{2}$$

Coupling with volumetric continuity, *Q=u*A*, into Eqs.(2), a full differentiation, in Eqs.(3), dealing with hydraulic force conservation could be accessed after transforming the variation of cross-section area into differential term. Appeal to the integral manipulation, an exact solution in Eqs.(4) will be easily lead out which might become as an alternative expression of so called modified Bernoulli's equation and is available to steady flow in pipe contraction or expansion.as the sketch in Fig.2. Here the parameter of variable pipe diameter has been taken into account. Additionally, a general definition, in Eqs.(5), related to the variation of static pressure along the streamline, is still required to enclosure the theoretic model.

$$-dPA + \rho d\frac{A}{2}u^2 + \frac{1}{2}\rho Q^2\left(\frac{1}{A}\right) = 0 \tag{3}$$

$$PA + \rho\frac{A}{2}u^2 + \frac{1}{2}\rho Q^2\frac{1}{A} = constant \tag{4}$$

$$P_{01} - P_{02} = \rho_\omega g\Delta h \tag{5}$$

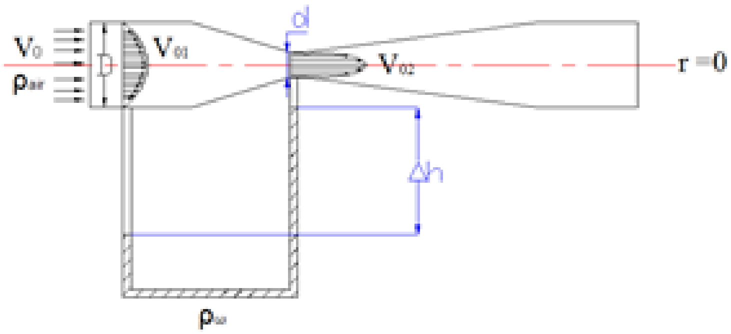

Fig. 2 The diagram of venture manometer

While substitute Eqs.(5) into Eqs.(4), a modified formula in Eqs.(6) demonstrates the relation among flow rate Q , scalar diameter as well as the difference of liquid elevated up at both tubes Δh . That will be used to correct the solution of Eqs.(7) widely used in previous model. Here A_i indicates the cross section area with pipe diameter at point i along the streamline and V_o is the flow velocity measured at the inlet of pipeline.

$$\Delta h = \frac{\rho Q^2}{\rho_\omega g A_2}\left(\frac{1}{A_2} - \frac{1}{A_1}\right) = \frac{\rho_a V_0^2}{\rho_\omega g}\left(\frac{D}{d}\right)^2\left[\left(\frac{D}{d}\right)^2 - 1\right] \tag{6}$$

$$\Delta h = \frac{\rho_a V_0^2}{2\rho_\omega g}\left[\left(\frac{D}{d}\right)^4 - 1\right] \tag{7}$$

3. EXPERIMENTAL PROCEDURE

To verify the validity of analytic solution introduced in this study, an auxiliary testing system and testing piezometers including static pressure tubes as well as dynamical pressure tube is set up in Fig.3~Fig.4. That features a special characteristic of smaller size, economical utility and efficient operation. Main components consisting of the experimental device are marked in Fig.3 designating (1) DC power supply, (2) sucking fan, (3) venturi-meter, (4) dynamical pressure tube , (5) static pressure tube and (6) dynamical testing device.

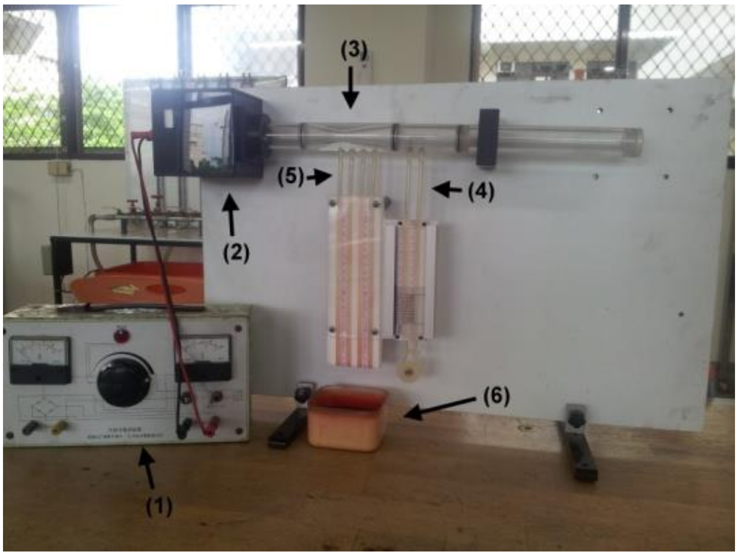

Fig. 3 Photograph of experimental mechanism

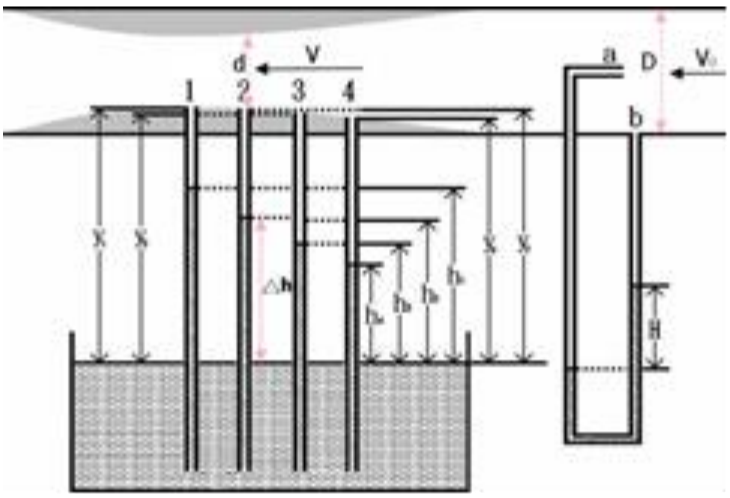

Fig. 4 The outlines of dynamical and static pressure gauge.

Proceeding to the experiment in progress, relevant testing procedure will be scheduled as follow.

1. Set up the testing mechanism as shown in Fig.3.
2. Turn on the power supply and regulate the input voltage 2V~30V by turns to create flow velocity of 5~12 m/s and records the rising value of liquid in individual tube while each voltage is imposed.
3. Estimate the inlet flow velocity using Δh measured at dynamical tube and evaluate corresponding flow rate.
4. Calculate the distribution of Δh at individual static tube based on the formulas given in Eqs.(6) and Eqs.(7) and compare the experimental results experienced in step 2.

4. RESULTS AND DISCUSSION

So far, our focus was concentrated on the development of theoretic model and experimental preparation. Next, we are going to figure out the distinction of Δh accessed from usage of various model. In Fig.6, the comparison was made under the flow velocity of 5.2 m/s specified at the inlet, and the position of constricted throat was located at the origin x=0 where the maximum flow velocity induced by the minimum negative pressure will come out. After then, the inner diameter in gradual expansion will be prescribed as x varies. Base on above arrangement, the fluid lifted up Δh for all profiles is found to have a decreasing intendancy as x increases, i.e., the lower value appears at larger inner diameter of pipe. Refer to flow continuity, such distributions seem to be nothing surprised due to the weaker negative pressure arisen by slower flow velocity induced inside the pipe of gradual expansion. Additionally, the solutions estimated from traditional formulas,1.5 cm ~0.1 cm, depicts a significant deviation from the measured results of 3.5 cm~0.1 cm while each test point is axially separately at the region of 0~4.5 cm. However, such unconformity could be further reduced by the assessment of modified model, fluid-elevation of 2.8 cm~0.1cm, deliveried at the same test region. Here the maximum relative error, 66%, calculated at throat using classical model has been effectively lessened to 20%.

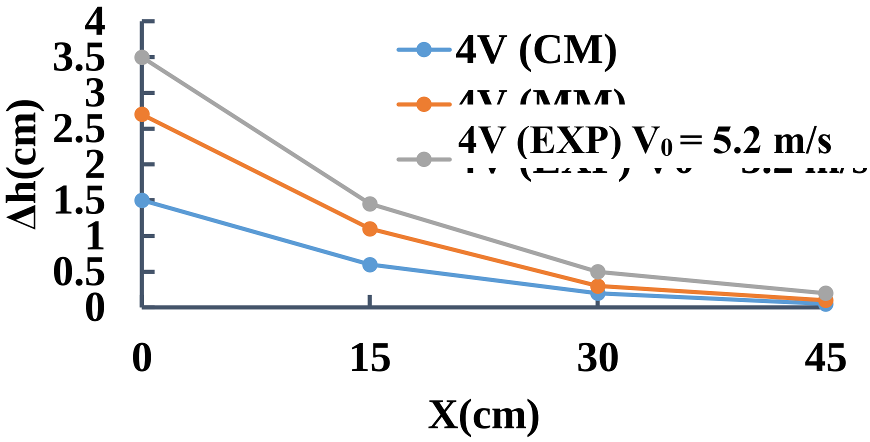

Fig. 5 Comparisons of Δh estimated from classical model (CM), modified model (MM) and experiment (EXP) while the inlet flow of 5.2 m/s is specified.

Similar to the discussion in Fig.5, both theoretic solutions and experimental results of Δh, experienced from the inlet flow velocity of 12.2 m/s, are plotted in Fig.6. Here the corrected data, compared to the outcomes fallen in Fig.5, is found to be augmented, i.e., classical solutions, 9 cm ~0.1 cm, yields the maximum relative error of 65% departing from experimental results of 22 cm ~0.1cm. Fortunately, such an relative error might be effectively overcome and dropped down to 20 % for the same interval of testing region ,while the evaluation ,with the resultant lifted magnitude of 0.1 cm~1.5 cm ,is made by modified model. View from quality concern, both discussions in Fig.5 and Fig.6are found to be compatible and which endorses the special advantage of proposed model by accounting the pipeline of varying diameter into developing process.

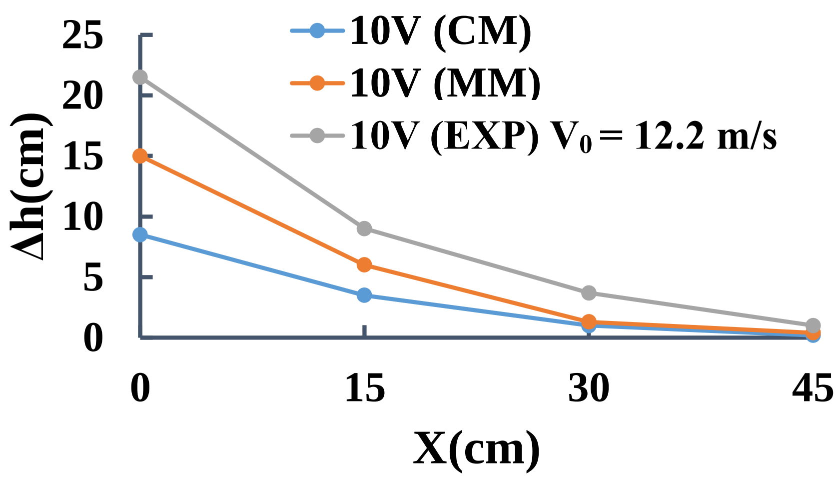

Fig. 6 Comparisons of Δh estimated from classical model (CM), modified model (MM) and experiment (EXP) while the inlet flow of 12.2 m/s is specified.

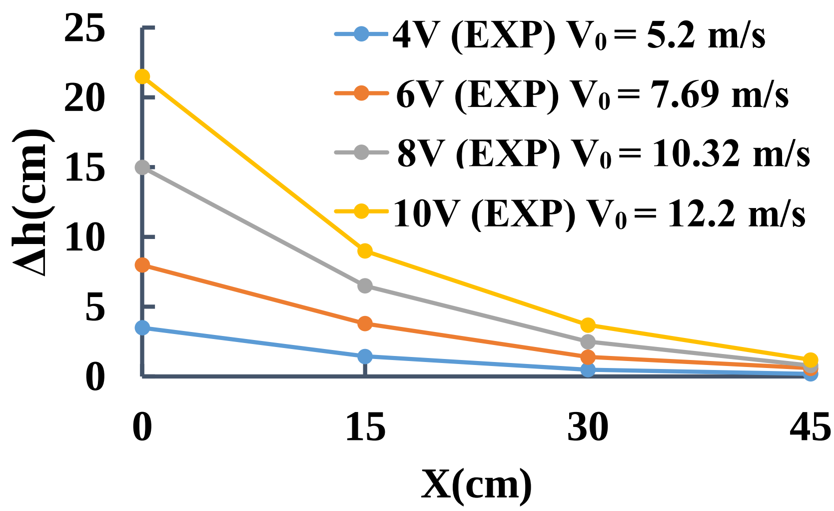

Fig. 7 Comparisons of experimental Δh estimated from the variation of inlet flow velocity.

Before we will enclosure the discussion, our interest will turn to understand the difference of hydraulic head, Δh, induced by various flow velocities. In Fig.7, inlet flow speed 5.2 m/s ~12.2 m/s will be generated by employing various voltages into sucking fan. Here the resultant profiles behave a decreasing intendancy as the air flows through the pipe conduit in gradual expansion. For example, individual experimental value with 22~0.1 cm, 15~0.1cm, 8~0.1 cm and 3.5 ~0.1 cm will be accessed as the measurement is made from x=0 (pipe throat) axially outward to x=4.5 cm, i.e., test region over tube 1 ~ 4 will be included as illustrated in Fig.4 with the flow at the pipe entrance rated as 5.2m/s, 7.7m/s, 10.3m/s and 12.2 m/s respectively. Besides, above dissemination also implies that the higher elevation Δh predicted in quicker flow suction seems not to be surprised due to the local negative pressure will be intensified under the faster flow velocity drawn at the entry of pipe in ambient pressure, and which has been clearly interpreted as the forgoing statement in Fig. 5.

5. CONCLUSIONS

Unlike previous Bernoulli's theory without accounting for the effect of piping contraction or expansion, the differential varied piping- volume approach, developed in this study, makes the proposed model more practical and suitable for engineering application. Besides the estimated errors might be well corrected, major hydraulic loss, determined by the differential Δh of experimental and empirical results, is found to be dependent on the evolution of turbulent vortex, which will be induced by discharge jet as air flow goes through the constricted throat into divergent portion. Summary from the magnitude of Δh distributed in Fig.6~Fig.7, an inversed approximation with the square of diameter might be generalized, that is, the expected maximum Δh occurs around the region of constricted throat where the strongest vortex intensity appears. In addition, the absence of frictional loss without losing global behavior could be also reconfirmed here since the length of pipeline isn't long enough considered in this study.

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