國　立　宜　蘭　大　學

九十九學年度轉學招生考試

（考生填寫）
准考證號碼：

微　積　分　試　題

《作答注意事項》

1. 請先檢查准考證號碼、座位號碼及答案卷號碼是否相符。
2. 考試時間：80 分鐘。
3. 本試卷共有單選擇 10 題，一題 10 分，不倒扣，共計 100 分。
4. 請將答案寫在答案卷上。（限用藍或黑色鋼筆、原子筆作答）
5. 考試中禁止使用大哥大或其他通信設備。
6. 考試後，請將試題卷及答案卷一併繳交。
7. 本試卷採雙面影印，請勿漏答。
8. 本試題附計算紙一張。

1. If $f(x)$ is a continuous function defined in R, and $\int_0^{x^2} f(t)dt = x\sin(\pi x)$, then $f(1)$ =

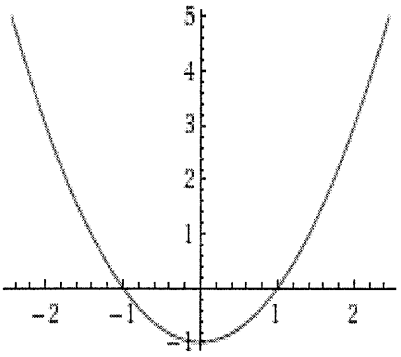
(A) 1 (B) $\frac{-\pi}{2}$ (C) 0 (D) -2

Hint: $\int_0^{x^2} f(t)dt = x\sin(\pi x)$

$\Rightarrow D\int_0^{x^2} f(t)dt = Dx\sin(\pi x)$

$\Rightarrow f(x^2)\cdot 2x = \sin(\pi x) + \pi x\cos\pi x$

2. Find the smallest value of the definite integral $\int_a^b (x^2-1)dx$ where $a<b$. (A) $\frac{-4}{3}$ (B) $\frac{-2}{3}$ (C) $\frac{2}{3}$ (D)It does not exist.

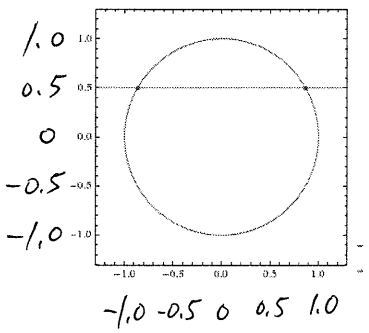


3. $\lim_{n\to\infty}\int_0^1 \frac{nx^{n-1}}{1+x}dx =$ (A) $\frac{1}{2}$ (B) 0 (C) 1 (D) ∞

Hint: $\int_0^1 \frac{nx^{n-1}}{1+x}dx = \int udv = uv - \int vdu = \frac{x^n}{1+x}\Big|_0^1 - \int_0^1 x^n\ln(1+x)dx$,

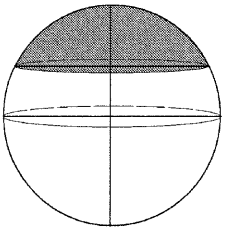
$0 \le \int_0^1 x^n\ln(1+x)dx \le \int_0^1 x^n(\ln 2)dx$

4. If $A=\{(x,y)\,|\,x^2+y^2\le 1\}$, $B=\left\{(x,y)\,\middle|\,y\ge\frac{1}{2}\right\}$, find the area of the region $A\cap B$. (A) $\frac{4\pi-3\sqrt{3}}{12}$ (B) $\frac{3\pi-2\sqrt{3}}{6}$ (C) $\frac{\pi}{6\sqrt{2}}$ (D) $\frac{\pi}{4\sqrt{2}}$.



5. If $A=\{(x,y,z)|x^2+y^2+z^2\le 1\}$, $B=\left\{(x,y,z)\middle|z\ge\frac{1}{2}\right\}$, find the volume of the solid $A\cap B$.

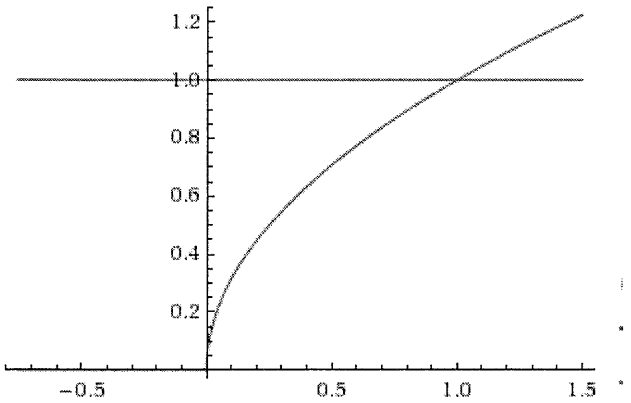
(A) $\frac{\pi}{6}$ (B) $\frac{5\pi}{24}$ (C) $\frac{\pi}{8}$ (D) $\frac{3\pi}{16}$.



6. $\int_0^1\int_{\sqrt{x}}^1\sqrt{1+y^3}\,dydx=$ (A) $\frac{2}{3}(2^{\frac{3}{2}})$ (B) $\frac{2}{3}(2^{\frac{3}{2}}-1)$ (C) $\frac{2}{9}(2^{\frac{3}{2}})$ (D) $\frac{2}{9}(2^{\frac{3}{2}}-1)$

Hint: $\{(x,y)|0\le x\le 1,\sqrt{x}\le y\le 1\}=\{(x,y)|0\le y\le 1,0\le x\le y^2\}$,

$\int_0^1\int_{\sqrt{x}}^1\sqrt{1+y^3}\,dydx=\int_0^1\int_0^{y^2}\sqrt{1+y^3}\,dxdy$



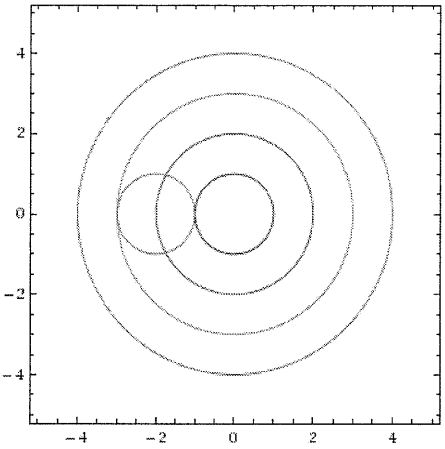
7. $\int_0^1\frac{x^3e^{x^2}}{(1+x^2)^2}dx=$ (A) $\frac{e-2}{6}$ (B) $\frac{e-1}{6}$ (C) $\frac{e-1}{4}$ (D) $\frac{e-2}{4}$.

Hint: $\int_0^1\frac{x^3e^{x^2}}{(1+x^2)^2}dx=\frac{1}{2}\int_0^1\frac{ue^u}{(1+u)^2}du=\frac{1}{2}\int_1^2\frac{(v-1)e^{v-1}}{v^2}dx$

$=\frac{1}{2e}\int_1^2\frac{(v-1)e^v}{v^2}dv=\frac{1}{2e}[\int_1^2(v^{-1}e^v-v^{-2}e^v)dv=\frac{1}{2e}[\int_1^2(v^{-1}e^v dv-\int_1^2 v^{-2}e^v dv]=$

$\frac{1}{2e}[v^{-1}e^v]\Big|_1^2+\int_1^2 v^{-2}e^v dv-\int_1^2 v^{-2}e^v dv=\frac{1}{2e}[v^{-1}e^v]\Big|_1^2=$

8. Find the maximum value of x^2+y^2 subject to the constraint $(x+2)^2+y^2=1$. (A) 8 (B)9 (C) 12 (D) 16

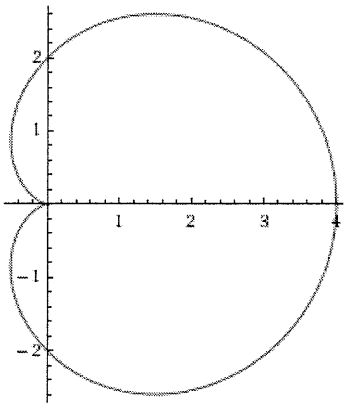


9. $\int_0^1 \frac{x^3-1}{\ln x}dx=$ (A) $\ln 3$ (B) $2\ln 3$ (C)$2\ln 2$ (D)$3\ln 2$.

Hint: $\int_0^1 \frac{x^3-1}{\ln x}dx=\int_0^1\int_0^3 x^y dydx=\int_0^3\int_0^1 x^y dxdy=\int_0^3 \frac{1}{y+1}x^{y+1}\Big|_0^1 dy=\int_0^3 \frac{1}{y+1}dy=$

10. Find the slope of the tangent line to the curve $r=2(1+\cos\theta)$ at the point $\theta=\frac{\pi}{2}$.

(A) -1 (B) 2 (C) 1 (D) -2



Hint: $\frac{dy}{dx}=\frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}}=\frac{\frac{d}{d\theta}(r\sin\theta)}{\frac{d}{d\theta}(r\cos\theta)}=\frac{r'\sin\theta+r\cos\theta}{r'\cos\theta-r\sin\theta}\Big|_{\theta=\frac{\pi}{2}}=\frac{r'}{-r}\Big|_{\theta=\frac{\pi}{2}}=\frac{2(-\sin\theta)}{-2(1+\cos\theta)}\Big|_{\theta=\frac{\pi}{2}}=$

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【計算紙】