國　立　宜　蘭　大　學

九十八學年度轉學招生考試

（考生填寫）
准考證號碼：

微　積　分　試　題

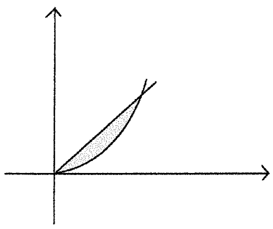
《作答注意事項》

1. 請先檢查准考證號碼、座位號碼及答案卷號碼是否相符。
2. 考試時間：80 分鐘。
3. 本試卷共有 15 題單選題，1-10 題每題 6 分，11-15 題每題 8 分，共計 100 分，答錯不倒扣。
4. 請將答案寫在答案卷上。（限用藍或黑色鋼筆、原子筆作答）
5. 考試中禁止使用大哥大或其他通信設備。
6. 考試後，請將試題卷及答案卷一併繳交。
7. 本試卷採雙面影印，請勿漏答。
8. 本試題附計算紙一張。

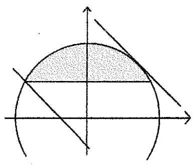
1. If $g(x)=x^2$, $f(x)=x+1$ then $(f\circ g)(x)=$(A) x^3+1 (B) x^3+x (C) x^2+1 (D). $(x+1)^2$ Hint: $(f\circ g)(x)=f(g(x))=f(x^2)=$
2. If $g(x)=x^2$ then $g'(1)=$(A) 1 (B) 2 (C) 3 (D) 4. Hint: $g'(x)=2x$
3. $\int_0^1 x dx=$ (A) $\frac{1}{2}$ (B) $\frac{1}{3}$ (C) 1 (D) $\frac{1}{4}$ Hint: $\int_0^1 x dx=\frac{1}{2}x^2\Big|_0^1$
4. $\int_0^\pi \cos x dx=$(A) 1 (B) 2 (C) -1 (D) 0. Hint: $\int_0^\pi \cos x dx=\sin x\Big|_0^\pi$
5. If $y=f(x)$ satisfies $x^2+y^2=2$, find $\frac{dy}{dx}\Big|_{\substack{x=1\\y=1}}$. (A) 1 (B) -1 (C) $\frac{1}{2}$ (D) $-\frac{1}{2}$

   Hint: $2x+2y\cdot y'=0$
6. Find the area of the region bounded by $y=x$ and $y=x^2$ (A) $\frac{1}{6}$ (B) $\frac{1}{4}$ (C) $\frac{1}{8}$ (D) $\frac{1}{2}$

   Hint: A= $\int_0^1 (x-x^2)dx=(\frac{1}{2}x^2-\frac{1}{3}x^3)\Big|_0^1=$



7. Find the minimum value of $x+y$ subject to the constraints $x^2+y^2\le 4$ and $y\ge 1$. (A) -2 (B) $-\sqrt{2}$ (C) $1-\sqrt{3}$ (D) $\frac{-1}{2}$ Hint: See the figure below.

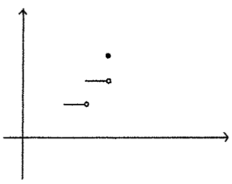


8. Find the interval of convergence of the power series $\sum_{i=0}^{\infty} x^i=1+x+x^2+x^3+\cdots$. (A) $\{0\}$

   (B) $(-1,1)$ (C) $[-1,1]$ (D) $(-\infty,\infty)$

   Hint: The power series $\sum_{i=0}^{\infty} x^i$ is a geometric series with ratio x.
9. Let $[x]$ be the greatest integer $\le x$. Find the Riemann integral $\int_2^4 [x]dx$. (A) 4 (B) 5

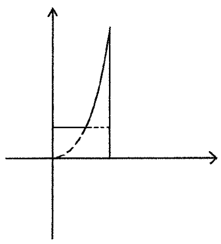
   (C) 6 (D) It does not exist.



背面尚有試題

10. Define $\max\{a,b\}:=\begin{cases} a, \text{ if } a \geq b \\ b, \text{ if } b \geq a \end{cases}$. Find $\int_0^2 \max\{1,x^2\}dx$. (A) 2 (B) $\frac{8}{3}$ (C) 3 (D) $\frac{10}{3}$

Hint: $\int_0^2 \max\{1,x^2\}dx = \int_0^1 1\,dx + \int_1^2 x^2\,dx$



11. Let $f: R \to R$ be a function defined by $f(x)=\begin{cases} 0, \text{ if } x \text{ is rational.} \\ 1, \text{ if } x \text{ is irrational.} \end{cases}$. Find the Riemann integral $\int_0^1 f(x)dx$. (A) 0 (B) 1 (C) $+\infty$ (D) It does not exist.

Hint: consider the lower sum and the upper sum.

12. If $f: R \to R$ is twice differentiable and $f''(x)$ is a nonzero function, then the equation $f(x)=0$ (A) may have one real root but no more. (B) may have two real roots but no more. (C) may have three real roots but no more. (D) may have infinitely many real roots.
Hint: $f''(x)$ is a positive or negative function.

13. Find the center of mass of the homogeneous solid (one eighth of the unit ball)
$S:=\left\{(x,y,z)\,\middle|\,x^2+y^2+z^2 \leq 1, x \geq 0, y \geq 0, z \geq 0\right\}$. (A) $(\frac{4}{3\pi},\frac{4}{3\pi},\frac{4}{3\pi})$ (B) $(\frac{3}{2\pi},\frac{3}{2\pi},\frac{3}{2\pi})$

(C) $(\frac{3}{8},\frac{3}{8},\frac{3}{8})$ (D) $(\frac{1}{3},\frac{1}{3},\frac{1}{3})$ Hint: $\bar{z} = \dfrac{\int_0^{\frac{\pi}{2}}\int_0^1\int_0^{\sqrt{1-r^2}} zrdzdrd\theta}{\frac{1}{8}\cdot\frac{4}{3}\pi}$

14. Let $f(x)=\sum_{i=0}^{4} a_i x^i$ be a fourth degree polynomial function with real- valued coefficients.
If $f(1)=f(-1)=1$, $f'(1)=-1$, $f'(-1)=1$, and $f''(1)=f''(-1)=0$, then (A) $a_3=0$

(B) $a_4=1$ (C) $a_1 \neq 0$ (D) $a_0=0$. Hint: $f''(x)=c(x-1)(x+1)=c(x^2-1)$

15. If $f(x)=\frac{1}{1-x^2}$, then the third derivative $f'''(0)=$ (A) 0 (B) 3! (C) 2! (D) 4!.

Hint: $\frac{1}{1-x^2}=1+x^2+x^4+x^6+\cdots$, $|x|<1$.

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