1. The set

$\{f \mid f \text{ is a continuous real-valued function defined on } [-1,1], \text{ and } |f(x)| = |x| \text{ for all } x \in [-1,1]\}$

has (A) just 1 element. (B) just 2 elements. (C) just 4 elements.
(D) infinitely many elements.

2. Let $f(x) = \frac{1}{6}x^6 - \frac{1}{5}x^5 + \frac{1}{30}$ be defined on the real line. How many real roots does the equation $f(x) = 0$ have? (A) It has just 1 real root. (B) It has just 6 real roots.
(C) It has no real root. (D) It has infinitely many real roots.

3. Let $f : R \to R$ be a function defined by $f(x^3) = \ln(x^2 + 1)$. Find $f\,'(8)$. (A) $\frac{1}{5}$ (B) $\frac{4}{5}$
(C) $\frac{1}{15}$ (D) $\frac{16}{65}$.

4. Suppose that $f : R \to R$ is a continuous function. If we define $g(x) := \int_0^x f(xt)dt$, find the right-hand derivative $g_+'(0)$. (A) 0 (B) $f(0)$ (C) $(f(0))^2$ (D) It may not exist.

5. $\int_0^{\frac{\pi}{2}} \cos^6 x dx =$ (A) 0 (B) 1 (C) $\frac{5\pi}{32}$ (D) $\frac{\pi}{12}$

6. $\lim_{x \to 0^+} \frac{\int_0^{3x} \frac{\sin(3t)}{t} dt}{x} =$ (A) 0 (B) 1 (C) 3 (D) 9

7. $\lim_{x \to 0^+} \frac{\int_1^{(1+x)^2} \frac{\sin(t-1)}{t-1} dt}{x} =$ (A) 0 (B) 2 (C) 1 (D) ∞

8. $\lim_{n \to \infty} \sum_{k=1}^{n} \frac{\ln(1+\frac{k}{n})}{n} =$ (A) $2\ln 2 + 1$ (B) $\ln 2$ (C) $2\ln 2$ (D) $2\ln 2 - 1$.

9. If $f(x) = x^5 + x^3 + x + 1$, then $(f^{-1})'(4) =$ (A) $\frac{1}{6}$ (B) $\frac{1}{7}$ (C) $\frac{1}{8}$ (D) $\frac{1}{9}$.

10. Find the curvature for the curve $\vec{\lambda}(t) = (\sin t, \cos t) + t(-\cos t, \sin t),\ t > 0$. (A) $\frac{1}{t}$ (B) $\frac{1}{\sqrt{t}}$
(C) $\frac{1}{t^2}$ (D) $\frac{1}{\sqrt[3]{t^2}}$

11. Find the maximum value of the function $f(x, y) = x^2 + xy + 2y^2$ subject to the constraint $x^2 + y^2 = 1$. (A) $\frac{3+\sqrt{2}}{2}$ (B) $\frac{4+\sqrt{2}}{2}$ (C) $\frac{4+\sqrt{3}}{2}$ (D) $\frac{3+\sqrt{3}}{2}$

12. Find the area of the surface $\left\{(x,y,z)\in R^3 \mid x^2+y^2+z^2=1,\ z\geq \frac{\sqrt{2}}{2}\right\}$. (A) $\frac{3}{2}\pi$ (B) $\frac{1}{2}\pi$

(C) $\sqrt{2}\pi$ (D) $(2-\sqrt{2})\pi$

13. $\int_0^1\int_{\sqrt{x}}^1 \sqrt{1+y^3}\,dydx =$ (A) $\frac{1}{9}(2^{\frac{3}{2}}-1)$ (B) $\frac{2}{9}(2^{\frac{3}{2}}-1)$ (C) $\frac{3}{4}(2^{\frac{3}{2}}-1)$ (D) $\frac{1}{4}(2^{\frac{3}{2}}-1)$

14. If $f(x,y,z)=g(r)$, where $r=\sqrt{x^2+y^2+z^2}$, then $\frac{\partial^2 f}{\partial x^2}+\frac{\partial^2 f}{\partial y^2}+\frac{\partial^2 f}{\partial z^2}=$

(A) $g''(r)+\frac{2g'(r)}{r}$ (B) $g''(r)+\frac{rg'(r)}{2}$ (C) $g'(r)+\frac{2g''(r)}{r}$ (D) $g'(r)+\frac{rg''(r)}{2}$.

15. Which of the following functions defined on R^2 is continuous at the origin $(0,0)$?

(A) $f(x,y)=\begin{cases}\frac{xy}{x^2+y^2} & \text{if } (x,y)\neq(0,0)\\ 0 & \text{if } (x,y)=(0,0)\end{cases}$ (B) $f(x,y)=\begin{cases}\frac{xy^2}{x^2+y^4} & \text{if } (x,y)\neq(0,0)\\ 0 & \text{if } (x,y)=(0,0)\end{cases}$

(C) $f(x,y)=\begin{cases}\frac{|xy|}{\sqrt{x^2+y^2}} & \text{if } (x,y)\neq(0,0)\\ 0 & \text{if } (x,y)=(0,0)\end{cases}$ (D) $f(x,y)=\begin{cases}\frac{\tan(x^2+y^2)}{x^2+y^2} & \text{if } (x,y)\neq(0,0)\\ 0 & \text{if } (x,y)=(0,0)\end{cases}$

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