

國 立 宜 蘭 大 學

1 0 5 學 年 度 研 究 所 碩 士 班 考 試 入 學

線性代數試題

(電 機 工 程 學 系 碩 士 班)

准考證號碼：

《作答注意事項》

- 1.請先檢查准考證號碼、座位號碼及答案卷號碼是否相符。
- 2.考試時間：100 分鐘。
- 3.本試卷共有 8 題，共計 100 分。
- 4.請將答案寫在答案卷上。
- 5.考試中禁止使用大哥大或其他通信設備。
- 6.考試後，請將試題卷及答案卷一併繳交。
- 7.應試時不得使用電子計算機。

1. Determine following matrices are in reduced echelon form, echelon form, or none of above. 12%

a. $\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix}$, b. $\begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$, c. $\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$, d. $\begin{bmatrix} 1 & 1 & 0 & 1 & 1 \\ 0 & 2 & 0 & 2 & 2 \\ 0 & 0 & 0 & 3 & 3 \\ 0 & 0 & 0 & 0 & 4 \end{bmatrix}$

2. Let \mathbf{A} be a 6×5 matrix. What must a and b be in order to define a linear transformation $T: \mathbb{R}^a \rightarrow \mathbb{R}^b$ by $T(\mathbf{x}) = \mathbf{A}\mathbf{x}$. 8%
3. The vector $\mathbf{x} = [2 \ 9 \ -7]^T$ is in a subspace H with a basis $B = \{[1 \ 4 \ -3]^T, [-2 \ -7 \ 5]^T\}$. Find the B-coordinate vector \mathbf{x} . 8%
4. Let $\mathbf{A} = \begin{bmatrix} -2 & 4 \\ -1 & 2 \end{bmatrix}$ and $\mathbf{w} = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$. Determine if \mathbf{w} is in Col \mathbf{A} . Is \mathbf{w} in Nul \mathbf{A} ? 8%
5. The set $B = \{1-t^2, t-t^2, 2-t+t^2\}$ is a basis for \mathbb{R}_2 . Find the coordinate vector of $\mathbf{p}(t) = 1+3t-6t^2$. 8%
6. If a 7×5 matrix \mathbf{A} has rank 2, find $\dim \text{Nul } \mathbf{A}$, $\dim \text{Row } \mathbf{A}$, and $\text{rank } \mathbf{A}^T$. 12%
7. Find the characteristic polynomial, eigenvalues of the matrix $\begin{bmatrix} 5 & 2 \\ 3 & 6 \end{bmatrix}$. Then, for each eigenvalue, find the corresponding eigenvector. 20%
8. Mark the following statement True or False and justify your answer.
- (a) The set $\text{Span}\{\mathbf{u}, \mathbf{v}\}$ is always visualized as a plane through the origin. 6%
 - (b) If $\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3$ are in \mathbb{R}^3 and \mathbf{v}_3 is not a linear combination of $\mathbf{v}_1, \mathbf{v}_2$, then $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$ is linearly independent. 6%
 - (c) The determinant of a triangular matrix is the sum of the entries on the main diagonal. 6%
 - (d) For an $m \times n$ matrix \mathbf{A} , if the equation $\mathbf{A}\mathbf{x} = \mathbf{b}$ is consistent, then Col \mathbf{A} is \mathbb{R}^m . 6%