

國立宜蘭大學

105 學年度研究所碩士班考試入學

工程數學(僅考線性代數與微分方程)試題

(電子工程學系碩士班)

准考證號碼：

《作答注意事項》

- 1.請先檢查准考證號碼、座位號碼及答案卷號碼是否相符。
- 2.考試時間：100 分鐘。
- 3.本試卷共有 14 題，單選題 11 題，一題 5 分，計算題 3 題，一題 15 分，共計 100 分。
- 4.請將答案寫在答案卷上。
- 5.考試中禁止使用大哥大或其他通信設備。
- 6.考試後，請將試題卷及答案卷一併繳交。
- 7.本試卷採雙面影印，請勿漏答。
- 8.應試時不得使用電子計算機。

Part I. 單選題 (每題五分, 答錯不倒扣)

1. The general solution of $(x+1)y' - (2x+3)y = 0$ is (A) $y = (c_1 + c_2x)e^x$ (B) $y = c(x+1)^2e^{-x}$ (C) $y = cx + x \ln x$ (D) $y = c(1+x)e^{2x}$ (E) $y = -1 + cx^3$
2. The general solution of $y'' + 4y' + 4y = 0$ is (A) $y = c_1 \cos(2x) + c_2 \sin(2x)$ (B) $y = (c_1 + c_2x)e^{-2x}$ (C) $y = c_1e^{2x} + c_2e^{-2x}$ (D) $y = c_1x + c_2x \ln x$ (E) $y = c_1x + c_2 \ln x$
3. Among the following differential equations, which one is **exact** (A) $x^2dy + 4y^2dx = 0$ (B) $\cos(x)dy + \sin(y)dx = 0$ (C) $\sin(x)dx + \cos(x)dy = 0$ (D) $x^2e^y y' + ye^x = 0$ (E) $\sin(3y)dx + 3x \cos(3y)dy = 0$
4. Which of the following is the Laplace transform of function t^2 (A) $\frac{1}{s^2}$ (B) $\frac{2}{s^2}$ (C) $\frac{1}{s^3}$ (D) $\frac{2}{s^3}$ (E) $\frac{1}{s}$
5. Which one is the solution of the partial differential equation $\frac{\partial u}{\partial x} - \frac{\partial u}{\partial y} = 0$? (A) $u(x, y) = Ce^{K(x+y)}$ (B) $u(x, y) = Ce^{K(k-y)}$ (C) $u(x, y) = Ce^{kxy}$ (D) $u(x, y) = Ce^{Kx/y}$ (E) $u(x, y) = Ce^{K(x^2+y^2)}$ (Here C and k are constants)
6. The inverse Laplace transform of the given function $\mathcal{L}^{-1}\left(\frac{2s+12}{s^2+16}\right) = ?$ (A) $2e^{-16x}$ (B) $2e^{-16x} + 12$ (C) $2e^{-4x} \sin 4x$ (D) $2e^{-4x} \cos 3x$ (E) $2\cos 4x + 3\sin 4x$
7. A period function $f(x)$ with period $T = 2\pi$ is defined as: $f(x) = \begin{cases} 1 & -\pi/2 < x < \pi/2 \\ 0 & \text{otherwise} \end{cases}$.
- This function can be represented by Fourier series $f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} (a_n \cos(nx) + b_n \sin(nx))$.
- $a_1 = ?$ (A) 0 (B) $\frac{1}{\pi}$ (C) $\frac{1}{2}$ (D) $\frac{2}{\pi}$ (E) $\frac{1}{2\pi}$

8. Given that $A = \begin{bmatrix} 1 & 1 & 1 \\ 2 & 3 & 0 \\ 3 & 5 & -1 \\ 4 & 7 & -2 \end{bmatrix}$. The rank of A equals (A) 0 (B) 1 (C) 2 (D) 3 (E) 4

9. Let $u = (1, 0, 1)$ and $v = (1, 1, 0)$. Which of the following vectors is not in $\text{Span}\{u, v\}$? (A) $(1, -1, 2)$ (B) $(4, 3, 1)$ (C) $(1, 1, 1)$ (D) $(1, 2, -1)$ (E) None of these.

10. Let $A = \begin{bmatrix} 2 & 0 & 1 & 6 \\ 0 & 3 & 2 & 0 \\ 0 & 8 & 0 & 0 \\ 0 & 9 & 0 & 5 \end{bmatrix}$, then $\det(A) = ?$ (A) 0 (B) -80 (C) 80 (D) -40 (E) 12

11. The set of vectors $\{(1, 2, 1), (0, 1, 1), (0, 1, x)\}$ is linear dependent. The variable $x = ?$ (A) -2 (B) -1 (C) 0 (D) 1 (E) 2

Part 2. 計算題 (每題十五分)

- Solve the differential equation $y'' + y' - 6y = 0$, $y(0) = 10$, $y'(0) = 0$
- Let $L: R^2 \rightarrow R^3$ be defined by

$$L(x) = \begin{bmatrix} x_1 + x_2 \\ x_1 - x_2 \\ 3x_1 + 2x_2 \end{bmatrix}$$

Find a matrix A such that $L(x) = Ax$ for each x in R^2

- Let

$$A = \begin{bmatrix} 3 & 2 \\ 3 & -2 \end{bmatrix}$$

Find all the eigenvalues and the corresponding eigenvectors.