國　立　宜　蘭　大　學

１０４學年度研究所碩士班考試入學

工程數學二試題

（電機工程學系碩士班）

准考證號碼：

《作答注意事項》

1.請先檢查准考證號碼、座位號碼及答案卷號碼是否相符。

2.考試時間：100 分鐘。

3.本試卷共有七題，共計 100 分。

4.請將答案寫在答案卷上。

5.考試中禁止使用大哥大或其他通信設備。

6.考試後，請將試題卷及答案卷一併繳交。

7.本試卷採雙面影印，請勿漏答。

8.應試時不得使用電子計算機。

1.(15%) In a certain lot of personal computers it is known that 1% have some minor defect as they come off the production line. They are put through a test procedure, which detects any defect 98% of the time if a defect is really present, and indicates a defect 1% of the time even though there is none present. What is the probability that a computer is

(a) defective if it fails the test; (8%)

(b) not defective if it did not fail the test? (7%)

2. (15%) Let X be a continuous random variable with probability density function (pdf)

$$f_X(x)=\begin{cases}2x & 0<x<1\\ 0 & otherwise\end{cases}$$

(a) Find the cumulative distribution function (cdf) $F_X(x)$. (4%)

(b) Find $P(\frac{1}{2}<X<2)$. (4%)

(c) Find the mean and variance of the random variable X. (7%)

3. (15%) A random variable X is uniformly distributed in the interval [-1, 1]. Find the pdf of the random variable $Y=X^2$.

4. (15%) If the probability mass function (pmf) of a random variable X is

$$P_X(x)=\begin{cases}0.3 & x=0\\ 0.7 & x=2\\ 0 & otherwise\end{cases}.$$

The conditional pmf for random variable Y given X is

$$P_{Y|X}(y\mid x=0)=\begin{cases}0.8 & y=0\\ 0.2 & y=1\\ 0 & otherwise\end{cases}\qquad P_{Y|X}(y\mid x=2)=\begin{cases}0.6 & y=0\\ 0.4 & y=1\\ 0 & otherwise\end{cases}$$

(a) Write the joint pmf $P_{XY}(x,y)$ as a table. (4%)

(b) What is the conditional pmf $P_{X|Y}(x\mid y=0)$? (4%)

(c) What is the conditional expectation $E[X\mid y=0]$? (3%)

(d) Let $Z=E[X\mid Y]$. Find the pmf of Z. (4%)

5. (15%) Let X_1 and X_2 be independent normal random variables, and
$X_1\sim N(mean, variance)=N(0,1)$, $X_2\sim N(0,1)$.
If $Z=X+Y$. Find the pdf of Z.

背面尚有試題

6. (10%) Consider a random variable X in which $P(X \geq 20) = 0.3$, $P(X \leq 10) = 0.3$, $E[X] = 15$. What is the lower bound for Var(X)?

7.(15%) The random variables X and Y are independent with probability density functions

$$f_X(x) = \begin{cases} \frac{8}{x^3} & x > 2 \\ 0 & otherwise \end{cases}$$

$$f_Y(y) = \begin{cases} 2y & 0 < y < 1 \\ 0 & otherwise \end{cases}$$

(a) Let $Z=XY$. Find the pdf of Z. (10%)

(b) Find $E[Z]$. (5%)