國　立　宜　蘭　大　學

102 學年度轉學招生考試

（考生填寫）
准考證號碼：

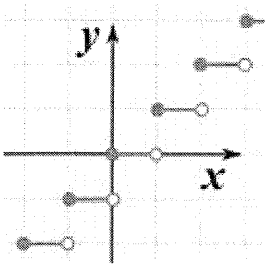
微　積　分　試　題

《作答注意事項》

1. 請先檢查准考證號碼、座位號碼及答案卷號碼是否相符。
2. 考試時間：80 分鐘。
3. 本試卷共有 20 題單選題，一題 5 分，不倒扣，共計 100 分。
4. 請將答案寫在答案卷上（於本試題上作答者，不予計分）。
5. 考試中禁止使用手機或其他通信設備。
6. 考試後，請將試題卷及答案卷一併繳交。
7. 本試卷採雙面影印，請勿漏答。

1. 設$[x]$表示小於或等於x之最大整數，試求$\lim_{z\to 1^+}[-z]$. (A) -2 (B)-1 (C)1 (D)2

Hint: Let $z = 1.1, 101, 1.001, \cdots$



2. $\lim_{x\to 0}\dfrac{\sqrt{1+x}-1}{x}=$ (A) 0 (B) 1 (C) $\dfrac{-1}{2}$ (D) $\dfrac{1}{2}$

Hint: $\lim_{x\to 0}\dfrac{(\sqrt{1+x}-1)\sqrt{1+x}+1)}{x(\sqrt{1+x}+1)}$

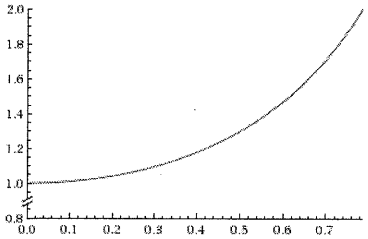
3. If $f(x)=\sin^3 x$, then $f'(\frac{\pi}{4})=$(A) $\dfrac{3\sqrt{2}}{4}$ (B) $\dfrac{-3\sqrt{2}}{4}$ (C) $\dfrac{3\sqrt{2}}{2}$ (D) $\dfrac{-3\sqrt{2}}{2}$

Hint: $Du^3 = 3u^2 Du = 3\sin^2 x\cdot\cos x$

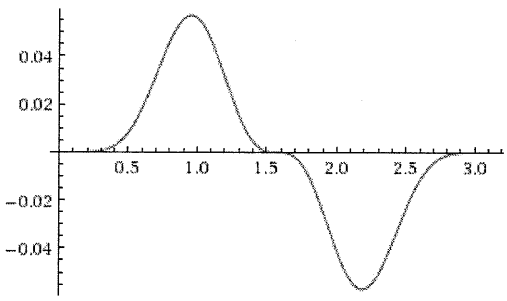
4. The solution of the equation $\begin{cases} x+y=1 \\ y+z=0 \\ x-z=1 \end{cases}$ in space is (A)two planes (B)a plane

(C) a point (D)a line . Hint: $(x+y=1)-(y+z=0)=(x-z=1)$

5. $\int_0^{\frac{\pi}{4}}\sec^2 x\,dx$ =(A) 0 (B) $\sqrt{2}$ (C) 1 (D) 2 Hint: $D\tan x=\sec^2 x$

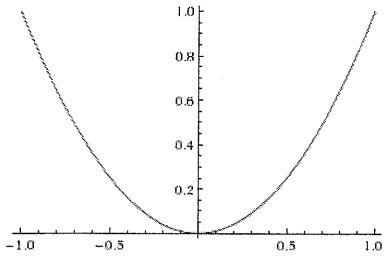


6. $\int_0^{\pi}\cos^3 x\cdot\sin^6 x\,dx=$ (A) $\dfrac{1}{9}$ (B) $\dfrac{1}{3}$ (C) 0 (D) $\dfrac{4}{9}$

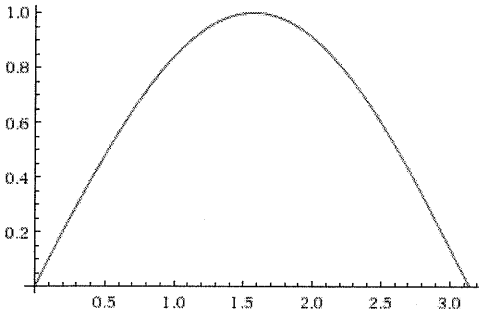


背面尚有試題

7. If $f(x)=ax^2+bx+c$, and $f(0)=0$, $f(\pm 1)=1$, then $a=$ (A) 1 (B) -1 (C) 0 (D) $\frac{1}{2}$



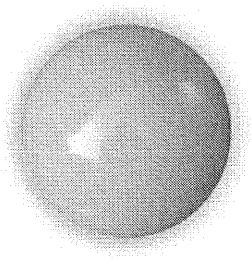
8. $\int_0^{\pi} \sin x\, dx =$ (A) 4 (B) 0 (C) -1 (D) 2



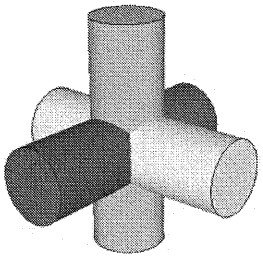
9. Let $f(x)=x$, find a number $c\in(0,1)$ such that $\int_0^1 f(x)dx = f(c)(1-0)$, $c=$ (A) $\frac{1}{3}$ (B) $\frac{1}{2}$ (C) $\frac{\sqrt{2}}{2}$ (D) $\frac{\sqrt{2}}{3}$

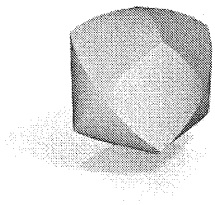
10. If S is the sphere of radius 1, then its surface area = (A) $\frac{4\pi}{3}$ (B) 4π (C) 2π (D) 1

Hint: $\int_0^{2\pi}\int_0^{\pi} \sin\phi d\phi d\theta = \int_0^{2\pi} 2d\theta =$



11. Let S be the solid common to three cylinders $x^2+y^2\le 1$, $x^2+z^2\le 1$, and $y^2+z^2\le 1$. Find the volume of S. (A) $\frac{16(\sqrt{2}-1)}{3}$ (B) $\frac{8\pi}{3}$ (C) $8(2-\sqrt{2})$ (D) $\frac{8\sqrt{2}}{3}$

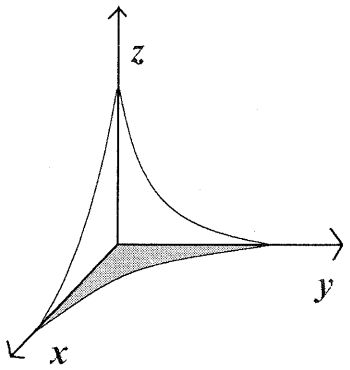


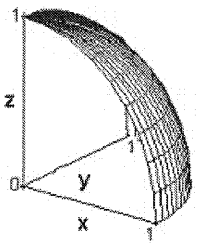


12. Let T be the solid enclosed by $x^{\frac{2}{3}}+y^{\frac{2}{3}}+z^{\frac{2}{3}}=1$ in the first octant. Find the volume of T.

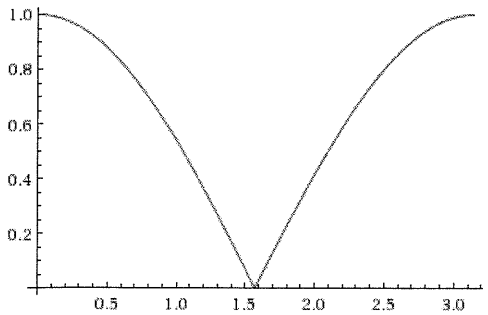
(A) $\frac{4\pi}{35}$ (B) $\frac{\pi}{70}$ (C) $\frac{2\pi}{35}$ (D) $\frac{3\pi}{70}$ Hint:

$$V=\iiint_S 27(uvw)^2\,dudvdw=\int_0^{\frac{\pi}{2}}\int_0^{\frac{\pi}{2}}\int_0^1 27(\rho\sin\phi\cos\theta\rho\sin\phi\sin\theta\rho\cos\phi)^2\rho^2\sin\phi d\rho d\phi d\theta$$





13. $\int_0^{\pi}|\cos x|\,dx=$ (A) 2 (B) 0 (C) $\sqrt{2}$ (D) $\sqrt{3}$



14. If $T=\{(x,y,z)\,|\,0\le x\le 1,0\le y\le x^2,0\le z\le x+y\}$, and the density of Q is given by

$\rho(x,y)=1$, find its mass. (A) $\frac{3}{20}$ (B) $\frac{5}{20}$ (C) $\frac{7}{20}$ (D) $\frac{9}{20}$

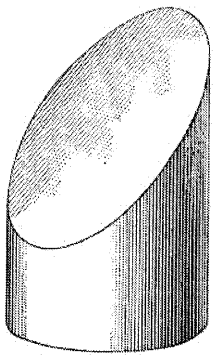
Hint: $M=\int_0^1(\int_0^{x^2}\int_0^{x+y}1dzdy)dx=\int_0^1\int_0^{x^2}(x+y)dydx$

15. $1+\frac{1}{2}+\frac{1}{3}\cdots=$ (A) $\ln 10$ (B) π (C) e (D) ∞ Hint: $\int_1^{\infty}\frac{1}{x}dx=\ln x\Big|_1^{\infty}=$

16. Define $\max\{a,b\}=\max\{b,a\}=a$ if $a\geq b$. Find $\int_0^{\pi}\max\{\cos x,0\}\,dx$. (A) 0 (B) 1 (C) π (D) $\frac{\pi}{2}$. Hint: $\int_0^{\pi}\max\{\cos x,0\}\,dx=(\int_0^{\frac{\pi}{2}}\cos x\,dx)+0$

17. Find the volume of the solid under the plane $z=x+y+3$ and over the unit disc $\{(x,y,0)\,|\,x^2+y^2\leq 1\}$. (A) 3π (B) 6π (C) 9π (D) $\frac{3\pi}{2}$

Hint: $\int_0^{2\pi}(\int_0^1(r\cos\theta+r\sin\theta+3)rdrd\theta=\int_0^{2\pi}(\int_0^1 3rdr)d\theta$

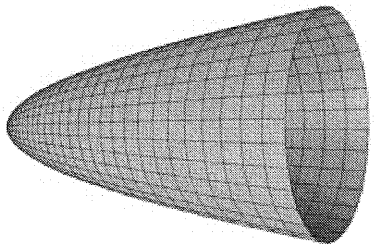


18. $\int_0^1\int_0^2\int_0^4 1\,dzdydx=$ (A) 8 (B) 7 (C) 0 (D) $7(x+y+z)$

19. Find the area of the surface generated by rotating $\gamma(t)=(t^2,3t)$, $t\in[0,4]$ about the x axis.

(A) $\frac{\pi}{2}(9^{\frac{3}{2}})$ (B) $\frac{\pi}{2}(73^{\frac{3}{2}})$ (C) $\frac{\pi}{2}(73^{\frac{3}{2}}-9^{\frac{3}{2}})$ (D) $\frac{\pi}{2}(73^{\frac{3}{2}}+9^{\frac{3}{2}})$

Hint: $A=\int_0^4 2\pi(3t)\sqrt{(2t)^2+(3)^2}\,dt=\frac{\pi}{2}(4t^2+9)^{\frac{3}{2}}\Big|_0^4=$



20. The number $0.3\overline{21}=$ (A) $\frac{318}{900}$ (B) $\frac{321}{900}$ (C) $\frac{321}{990}$ (D) $\frac{318}{990}$.