1. (a) State the stability condition of a linear system with the following characteristic equation,

$$s^4 + 3s^3 + 3s^2 + 3s + 2 = 0 \text{ . (15\%)}$$

(b) Find the range of k such that the linear system with the characteristic equation,

$$s^3 + 3ks^2 + (k+2)s + 4 = 0 \text{ ,}$$

is stable. (10%)

2. (a) Let the transfer function of the plant in a unit feedback control system be

$$G(s) = \frac{3(1+.5s)}{(1+s)(1+2s)(1+s+s^2)} \text{ .}$$

Find the steady-state error of the unit step response. (10%)

(b) Design a controller $G_c(s)$ to eliminate the steady-state error in (a), and plot the block diagram for the control system you designed. (15%)

3. (a) For a closed loop control system with the state equation

$$\dot{\mathbf{X}} = \mathbf{AX} \quad \text{and} \quad \mathbf{A} = \begin{bmatrix} 1 & -1 & -1 \\ 1 & 3 & 1 \\ -3 & 1 & -1 \end{bmatrix}$$

Determine if this control system is stable and explain the reason. (10%)

(b) Represent the control system in (a) with a differential equation, and solve it to support your answer in (a). (10%)

4. Consider the input-output transfer function:

$$H(s) = \frac{2s^2 + s + 5}{s^3 + 6s^2 + 11s + 4}$$

(a) Find the dynamic equations of the system in the controllable canonical form. (10%)
(b) Find the dynamic equations of the system in the observable canonical form. (10%)

5. Write an example transfer function for each of the following controllers.
(a) PID controller (5%)
(b) Phase-lag controller (5%)