

Part I. 單擇題（共30分，每題五分、答錯倒扣一分）

1. The general solution of $y'' + 2y' = 4x$, is (A) $c(1+x)e^{2x}$ (B) $2x-1+c_1e^{-2x}$ (C)

$$y = 4e^{-2x}(\cos 2x + \sin 2x) \quad (D) \quad x^2 - x + c_1e^{-2x} + c_2 \quad (E) -1 + cx^3.$$

2. Let $M(f)$ be the Fourier transform of $m(t)$, the Fourier transform of $m(2t)\cos(2\pi \cdot f_c t)$ is

$$(A) \frac{1}{4}[M(\frac{f}{2} + f_c) + M(\frac{f}{2} - f_c)] \quad (B) \frac{1}{2}[M(f + f_c) + M(f - f_c)] \quad (C) M(2f)\cos(2\pi \cdot f_c f)$$

$$(D) [M(2f + f_c) - M(2f - f_c)] \quad (E) 2[M(2f + f_c) + M(2f - f_c)].$$

3. Which of the following transformations is not a linear mapping?

$$(A) d/dx : \mathfrak{R}[x] \rightarrow \mathfrak{R}[x], \text{ where } d/dx \text{ is the differential operator.} \quad (B) T(x_1, x_2) = (1 + x_1, x_2)$$

$$(C) T : \mathfrak{R}^{m \times n} \rightarrow \mathfrak{R}, T(X) = \det(X) \quad (D) T(x_1, x_2) = (x_1 - x_2, 0)$$

$$(E) T : \mathfrak{R}^{m \times n} \rightarrow \mathfrak{R}^{n \times m}, T(X) = X^T.$$

4. Which of the following statements is not true?

(A) If A is nonsingular, then A and the identity matrix I are row equivalent.

$$(B) AB = 0 \text{ then } |A| = 0 \text{ or } |B| = 0$$

(C) $\text{Rank}(AB) \geq \text{Rank}(A)$ for any $n \times n$ matrix A, B .

(D) If eigenvalues $\lambda_1 \neq \lambda_2$, the corresponding eigenvectors x_1, x_2 are linear dependent.

(E) If A is nonsingular, then $\dim(\ker(A)) = 0$.

$$5. \text{ If } A = \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix} \text{ and } \det(A) = -2, \text{ then } \det \left(3 \begin{bmatrix} d+2g & e+2h & f+2i \\ a & b & c \\ -2g & -2h & -2i \end{bmatrix} \right) = ? \quad (A) 6$$

$$(B) 12 \quad (C) -12 \quad (D) 24 \quad (E) -108.$$

6. The eigenvalues of $A = \begin{bmatrix} -4 & -6 \\ 3 & 5 \end{bmatrix}$ are (A) 1, 2 (B) 1, -2 (C) -1, 2 (D) -1, -2 (E) 0, 2.

※ 注意：請在答案卷上作答，寫在試題卷之答案不予採計。

Part II. 計算題 (共70分，每題10分)

1. Solve the differential equation $x(x-1)y''+(3x-1)y'+y=0$.

2. Solve
$$\begin{cases} x'+y'-x-2y=0 \\ x'+2y'-5x-7y=e^{-t} \end{cases}$$
.

3. Find the general solution of P.D.E. $\frac{\partial u(x,y)}{\partial x} + \frac{\partial u(x,y)}{\partial y} = 0$

4. Find $L^{-1}\left\{\frac{s}{s^2+4s+20}\right\}$ and $L^{-1}\left\{\frac{s^2+2s+5}{(s^2+2s-3)^2}\right\}=?$.

5. Solve the following system using the method of LU decomposition.

$$\begin{cases} x_1 + x_2 + 2x_3 = 8 \\ 2x_1 + x_2 + 3x_3 = 13 \\ 3x_1 + 3x_2 + 7x_3 = 26 \end{cases}$$

6. Use the Gram-Schmidt process to construct an orthonormal set from the following three

vectors: $(1, 1, 1, 1)$, $(-1, 4, 4, 1)$, and $(4, -2, 2, 0)$.

7. Find the basis of $\text{kernel}(T)$ and $\text{Range}(T)$ for a linear transformation given by

$$T(x) = \begin{bmatrix} 1 & 2 & 3 & 4 \\ 1 & 3 & 5 & 7 \\ 1 & 0 & -1 & -2 \end{bmatrix} x.$$