1. (20%) Determine the condition on b_1, b_2, d_1, and d_2, so that the following system is controllable and observable.

$$\frac{dx(t)}{dt} = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} x(t) + \begin{bmatrix} b_1 \\ b_2 \end{bmatrix} u(t)$$
$$y(t) = \begin{bmatrix} d_1 & d_2 \end{bmatrix} x(t)$$

2. (20%) A controlled process is represented by the following dynamic equations:

$$\frac{dx_1(t)}{dt} = -x_1(t) + 5x_2(t)$$
$$\frac{dx_2(t)}{dt} = -6x_1(t) + u(t)$$
$$y(t) = x_1(t)$$

The control is obtained through state feedback with

$$u(t) = -k_1 x_1(t) - k_2 x_2(t) + r(t)$$

where k_1 and k_2 are real constants, and $r(t)$ is the reference input.

(a) Find the value of k_1 and k_2 such that ξ=0.707 and ω_n=10 rad/sec.

(b) Let the error signal be defined as $e(t) = r(t) - y(t)$. Find the steady-state error when $r(t) = u_s(t)$ and k_1 and k_2 are at the values found in part (a).

3. (20%) For the characteristic equation is given,

$$s^4 + 12.5s^3 + s^2 + 5s + K = 0$$

(a) Determine the range of K so that the system is stable.

(b) Determine the value of K so that the system is marginally stable and the frequency of sustained oscillation if applicable.

4. (20%) Construct the root locus for $K \geqq 0$

$$G(s) = \frac{K(s+8)}{s(s+5)(s+6)}$$

5. (20%) The transfer function of a unity feedback control system is

$$G(s) = \frac{K}{s(s+2)(s+10)}$$

(a) Using Nyquist criterion, determine the range of K such that the closed-loop system is stable.

(b) If K=1, find the gain margin and the phase crossover frequency of the system.