1. (5%) Let $Q(x, y)$ be the statement "$x+y=x-y$." If the domain for both variables consists of all integers, what are their truth values?
   (a) $Q(1,1)$
   (b) $\exists x\, Q(x,2)$
   (c) $\exists y \forall x Q(x,y)$
   (d) $\forall y\, \exists x\, Q(x,y)$
   (e) $\forall x \forall y Q(x,y)$.

2. (7%) Let n be an integer. Prove that if n^3+5 is odd, then n is even. (Hint: using a proof by contraposition)

3. (8%) Let S be a set. If there are exactly n distinct elements in S where n is a nonnegative integer, we say that n is the cardinality of S. What is the cardinality of each of these sets?
   (a) {5}
   (b) {{},{5}}
   (c) {5, {5}}
   (d) {5, {5},{5, {5}}}

4. (12%) Determine whether each of these functions is a bijection (one-to-one and onto) from R to R. Discuss the correctness of your answers.
   (a) $f(x)=2x+1$
   (b) $f(x)=x^2+1$
   (c) $f(x)=x^3$
   (d) $f(x)=(x^2+1)/(x^2+2)$

5. (7%) Show that at least 19 cables are needed for connecting eight computers to four printers such that four computers can directly access four different printers at the same time.

6. (6%) What is the probability of these events when we randomly select a permutation of {1, 2, 3}?
   (a) 1 precedes 3.
   (b) 3 precedes 1 or 3 precedes 2.
   (c) 3 precedes 1 and 3 precedes 2.

7. (10 %) Determine whether the relation R on the set of all integers has reflexive property, symmetric property, and transitive property. Discuss the correctness of your answers. Note that a relation R can have more than one property.
   (a) $(x,y)\in R$ if and only if $x\neq y$
   (b) $(x,y)\in R$ if and only if $xy\geq 1$
   (c) $(x,y)\in R$ if and only if $x\equiv y \pmod 7$
   (d) $(x,y)\in R$ if and only if x is a multiple of y
   (e) $(x,y)\in R$ if and only if $x\geq y^2$

8. (16 %) The notations for several classes of simple graphs are given as follows:
   Kn: the complete graph on n vertices, where $n \geq 2$. It is the simple graph that contains exactly one edge between each pair of distinct vertices.
   Cn: the cycle of n vertices, where $n \geq 3$. It consists of vertices $v_1, v_2,\ldots,v_n$, and edge $\{v_1,v_2\}$, $\{v_2,v_3\},\ldots,\{v_{n-1},v_n\}$ and $\{v_n, v_1\}$.
   Wn: the wheel of n+1 vertices. It is formed by adding an additional vertex to the cycle *Cn* and connecting this new vertex to each of the n vertices in *Cn*.
   Qn: the n-dimensional hypercube.
   Based on the above definitions, answer the following questions.
   (a) Give the number of edges for *Kn*.
   (b) Give the number of edges for *Cn*.
   (c) Give the number of edges for *Wn*.
   (d) Give the number of edges for *Qn*.
   (e) For which value of n is *Kn* bipartite?
   (f) For which value of n is *Cn* bipartite?
   (g) For which value of n is *Wn* bipartite?
   (h) For which value of n is *Qn* bipartite?

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9. (12 %) A rooted tree is called an m-ary tree if every internal vertex has no more than m children. The tree is called a full m-ary tree if every internal vertex has exactly m children. An m-ary tree with m=2 is called a binary tree. Please answer the following questions.
   (a) How many edges does a tree with 1,000 vertices have?
   (b)How many vertices does a full 5-ary tree with 1,000 internal vertices have?
   (c)How many edges does a full binary tree with 1,000 internal vertices have?
   (d) How many leaves does a full 3-ary tree with 1,000 vertices have?

10. (7%) Find a recurrence relation and give initial conditions for the number of bit strings of a length n that do not have two consecutive 0s. How many such bit strings are there of length eight?

11. (5 %) Solve the recurrence relation $f(n)=2f(n/2)+n$, where $n=2^k$ and k is a positive integer.

12. (5 %) Find the language recognized by the given deterministic finite-state automaton.