

Part I. 單擇題 (共30分，每題五分、答錯倒扣一分)

1. The solution of $y(x) + \int_0^x y(\tau) d\tau = 1$ is (A) $(1 - \frac{1}{2}x)e^x$ (B) e^{-x} (C) $x + x \ln x$
(D) $(1+x)e^{2x}$ (E) $-1+x^3$.
2. Let $\mathcal{L}[f(t)] = \left\{ \frac{2s^2 + 1}{s^3 + 6s^2 + 11s + 6} \right\}$, then (A) $f(0) = 0$ (B) $f(0) = 1$ (C) $f(0) = 2$
(D) $\lim_{t \rightarrow \infty} f(t) = -1$ (E) $\lim_{t \rightarrow \infty} f(t) = 1$.
3. Let $T = T_2 \circ T_1$ and $T_1(x, y) = (y, 2x + y, -x)$ $T_2(x, y, z) = (-x, x + y, 2y - z)$, the image of $(-2, 5)$ under T is (A) $(5, 1)$ (B) $(2, 3)$ (C) $(5, 3, 8)$ (D) $(-5, 6, 0)$ (E) not exist.
4. Determine which of the following statements is false for square matrices A, B of the same size.
(A) If $|A - B| = 0$ then $|A| = |B|$. (B) $AB = 0$ then $|A| = 0$ or $|B| = 0$.
(C) $|A^2| = (|A|)^2$ (D) If A is nonsingular, it is row equivalent to the identity matrix.
(E) If a diagonal matrix A is singular then at least one diagonal element is zero..
5. Let T be a linear transformation from V to W , Which of the following statements is false?
(A) if $\dim(V) = \dim(\text{Range}(T)) = 2$, then $\dim(\ker(T)) = 0$.
(B) if $\dim(V) = 5$ and $\text{rank}(T) = \dim(W) = 3$, then $\text{nullity}(T) = 2$.
(C) if $\dim(\ker(T)) = \dim(\text{Range}(T)) = 2$, then $\dim(V) = 4$.
(D) if $\dim(V) = 5$ and $\dim(W) = 3$, then $\text{rank}(T) = 2$.
(E) if $\dim(V) = 5$, $\dim(W) = 1$, and $T \neq 0$, then $\text{nullity}(T) = 4$.
6. Suppose A is a 4×4 matrix with eigenvalues 1, 2, 3, 4, then which one is not an eigenvalue of $2A^{-1} + I$. (A) $\frac{3}{2}$ (B) $\frac{5}{3}$ (C) 3 (D) 2 (E) 1.

※ 注意：請在答案卷上作答，寫在試題卷之答案不予採計。

(計算題於下一頁)

背面尚有試題

Part II. 計算題 (共70分，每題10分)

1. Solve the differential equation $(D^2 - D + 2)y = e^x \sin x$.

2. Find the convolution of $f(t) = u(t) - u(t-1)$ and $g(t) = 0.5[u(t) - u(t-1)]$.

3. Find the Fourier integral of $f(x) = \begin{cases} 1, & |x| < 1 \\ 0, & |x| \geq 1 \end{cases}$.

4. Solve $4x^2y'' + 4xy' - y = \frac{12}{x}$.

5. Let $A = \begin{bmatrix} \frac{4}{5} & \frac{1}{5} \\ \frac{1}{5} & \frac{4}{5} \end{bmatrix}$, Find (a) the eigenvectors of A , and (b) $\lim_{n \rightarrow \infty} A^n$.

6. Let $B = \begin{bmatrix} 0 & 1 & -1 & -2 \\ 2 & 0 & 0 & 0 \\ 0 & 2 & -3 & -5 \\ 0 & -1 & 3 & 5 \end{bmatrix}$. Find B^{-1} and $\det(B)$.

7. Find the Q-R decomposition of $A = \begin{bmatrix} 3 & -1 & 1 \\ 0 & 0 & -10 \\ 4 & 7 & -7 \end{bmatrix}$.