1.Solve the differential equation in following.

$$y^2 + x^2 \frac{dy}{dx} = xy \frac{dy}{dx}$$ (10%)

2. Solve the differential equation in following. (10%)

$$y' + y = (xy)^2$$

3.Find the inverse Laplace transforms of following equation. (10%)

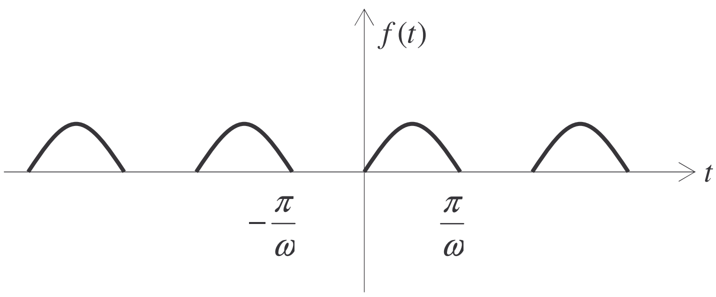
$$F(s) = \frac{s+1}{(s^2+4s+13)(s^2+4s+3)}$$

4.Solve the following equation using Laplace transforms. (10%)

$$y'' + y' = g(t) \quad , \quad g(t) = \begin{cases} 0 & , \quad 0 \le t \le 2 \\ 2 & , \quad t > 2 \end{cases} \quad , \quad y(0) = y'(0) = 0$$

5.(1). A sinusoidal voltage $2\sin \omega t$, where t is time, is passed through a half-wave rectifier that clips the negative portion of the wave ($fig.1$). Find the Fourier series of the resulting periodic function $f(t) = \begin{cases} 0 & \text{if} \quad -\frac{\pi}{\omega} < t < 0, \\ 2\sin \omega t & \text{if} \quad 0 < t < \frac{\pi}{\omega} \end{cases}$ (10%)

(2).Using (1) to evaluate $\frac{1}{1\times 3} + \frac{1}{3\times 5} + \frac{1}{5\times 7} + \cdots = ?$ (5%)



$fig.1$

6.Prove Cauchy Integral Formula. Let $f(z)$ be differentiable on an open set G. Let C be a closed path in G enclosing only points of G. Then, for any z_0 enclosed by G,

$$f(z_0) = \frac{1}{2\pi i}\oint_c \frac{f(z)}{z - z_0}dz \quad .$$ (15%)

7.(1). Evaluate the following integral by Residue theorem.

$$\int_{-\infty}^{\infty}\frac{1}{(s-2)^2(s^2+9)}ds = ?$$ (10%)

(2). Use residue theorem to evaluate the inverse Laplace transform of $\frac{1}{(s-2)^2(s^2+9)}$. (10%)

8.Given $A = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$ (10%)

(1). Find $e^A = ?$

(2). Find $\cos A = ?$