1.Solve each of the differential equations in following.

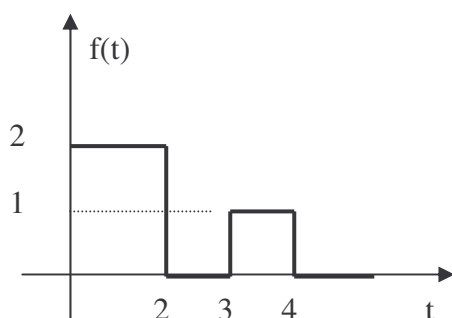　　(a)　$xy' = x^{-1}y^2 + y$ (10%)

　　(b)　$y'' - 8y' + 16y = 8\sin 2x$ (10%)

2.Find the orthogonal trajectories of the curves   (10%)

$$y = \frac{1}{2}x^2 + 3$$

3.Find the Laplace transforms$\cdots\{$f (t)$\}$ for the given $f(t)$.   (10%)

4.Solve the initial-value problem in following.   (10%)

　　$y'' + y' - 2y = 5t + e^{2t}$ ， $y(0) = y'(0) = 1$

5.Evaluate $\oint_c e^{1/z}\, dz$, for c any closed path not passing through the origin.   (10%)

6.Let A be a square matrix such that $A^{-1} = A^t$. Prove that $|A| = \pm 1$.   (10%)

7.Given $f(x) = xe^{-|x|}$,

　(a) Find the Fourier integral representation of $f(x)$.   (8 %)

　(b) Evaluate $\int_0^\infty \frac{\omega \cdot \sin(\omega)}{(1+\omega^2)^2}\, d\omega$, using the results of (1).   (7 %)

8.Use residue theorem to evaluate the inverse Laplace transform of $\dfrac{1}{\sqrt{s+1}}$.

　(Hint：$\int_0^\infty e^{-x^2}\, dx = \dfrac{\sqrt{\pi}}{2}$ & $\int_0^\infty e^{-ax^2}\, dx = \dfrac{\sqrt{\pi}}{2\sqrt{a}}$)   (15 %)