Part I 選擇題 (共 40%，每題五分，答錯倒扣一分)

1. The general solution of a differential equation $dy-(1+y^2)dx=0$ is (A) $x=\sec(y+c)$ (B) $y=\sec(x+c)$ (C) $x=\tan(y+c)$ (D) $y=\tan(x+c)$ (E) $y=\cos(x^2+c)$.

2. If the inverse Laplace transform of $\dfrac{2s}{(s-3)^2+4}$ is $e^{mt}(a\cos wt+b\sin wt)$, then (A) $m=4$ (B) $a=2$ (C) $b=-1$ (D) $w=3$ (E) $w=4$.

3. The Fourier transform of $f(t)$ is defined as $F(w)=\dfrac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty} f(t)e^{-jwt}dt$. Here, $f(t)=\begin{cases} k, & 0<t<1 \\ 0, & \text{otherwise} \end{cases}$. If the Fourier transform of $f(t)$ is $F(w)=\dfrac{k(1-e^{-jcw})}{jwd}$, then (A) $c=2$ (B) $c=1$ (C) $d=\sqrt{\pi}$ (D) $d=\sqrt{2}$ (E) $d=3$.

4. Which one is the solution of the partial differential equation $\dfrac{\partial V}{\partial x}+\dfrac{\partial V}{\partial y}=(x+y)V$? (A) $V(x,y)=Ce^{\left[\frac{1}{2}(x^2-y^2)+k(x-y)\right]}$ (B) $V(x,y)=Ce^{\left[\frac{1}{2}(x^2+y^2)+k(x+y)\right]}$ (C) $V(x,y)=Ce^{\left[\frac{1}{2}(x^2-y^2)+k(x+y)\right]}$ (D) $V(x,y)=Ce^{\left[\frac{1}{2}(x^2+y^2)+k(x-y)\right]}$ (E) $V(x,y)=Ce^{\left[x^2+y^2+k(x+y)\right]}$ (Here, C and k are constants. You can solve this problem by separating variables.)

5. Which of the following collections of vectors has a different vector space
   (A) $(1,2,4)^T$, $(2,1,3)^T$, $(4,-1,1)^T$
   (B) $(1,2,4)^T$, $(3,3,7)^T$, $(5,1,5)^T$
   (C) $(3,0,2)^T$, $(0,3,5)^T$,
   (D) $(1,-1,-1)^T$, $(2,1,3)^T$, $(5,1,5)^T$
   (E) $(1,2,4)^T$, $(1,1,3)^T$, $(3,0,2)^T$

6. Which of the following collections of vectors is linear dependent
   (A) $(1,1,1)^T$, $(1,1,0)^T$, $(1,0,0)^T$
   (B) $(1,2,4)^T$, $(2,1,3)^T$, $(4,-1,2)^T$
   (C) $(1,0,1)^T$, $(0,1,0)^T$
   (D) $p_1(x)=x^2-2x+3$, $p_2(x)=2x^2+x+8$ $p_3(x)=x^2+8x+7$
   (E) $\begin{bmatrix}1&0\\0&1\end{bmatrix}, \begin{bmatrix}0&0\\0&1\end{bmatrix}, \begin{bmatrix}0&1\\0&1\end{bmatrix}, \begin{bmatrix}1&0\\1&0\end{bmatrix}$

7. If $A=\begin{bmatrix}1 & 2 & 1 & 1\\ 2 & -1 & 1 & 2\\ 4 & 3 & 3 & 4\\ 3 & 1 & 2 & 3\end{bmatrix}$, the rank of A equals (A)1 (B)2 (C) 3 (D) 4 (E) 0

8. Let $A=\begin{bmatrix}2 & 1 & 0 & 0\\ 0 & 2 & 0 & 0\\ 20 & 20 & 3 & 1\\ 20 & 20 & 1 & 1\end{bmatrix}$, then $\det(A)=?$ (A) 0 (B) 2 (C) 4 (D) 8 (E) 10

Part II 計算題 (60%，每題十分)

1. Solve the differential equation $y'-4y=e^x-\sin x$.
2. To solve the initial-value problem $y''+2y'+y=2\delta(t-1),\ y(0)=1,\ y'(0)=1$.
3. A periodic function $f(t)$ with period $T=2\pi$ is defined as $f(t)=|t|, -\pi\le t\le\pi$. This function can be represented by a Fourier series: $f(t)=\frac{a_0}{2}+\sum_{n=1}^{\infty}(a_n\cos nt+b_n\sin nt)$. Find a_0, a_n and b_n.
4. Find the eigenvalues and eigenvectors of the matrix given by

$$A=\begin{bmatrix}2 & -3 & 1\\ 1 & -2 & 1\\ 1 & -3 & 2\end{bmatrix}.$$

5. Let

$$A=\begin{bmatrix}1 & -1 & 4\\ 1 & 4 & -2\\ 1 & 4 & 2\\ 1 & -1 & 0\end{bmatrix}$$

Find an orthonormal basis for the column space of A.

6. Let $L: R^2\to R^3$ be defined by

$$L(x)=\begin{bmatrix}x_1+x_2\\ x_1-x_2\\ 3x_1+2x_2\end{bmatrix}.$$

Find a matrix A such that $L(x)=Ax$ for each x in R^2.