

國立宜蘭大學

106 學年度研究所碩士班考試入學

工程數學(僅考矩陣與一、二階微分方程)試題

(電子工程學系碩士班)

准考證號碼：

《作答注意事項》

- 1.請先檢查准考證號碼、座位號碼及答案卷號碼是否相符。
- 2.考試時間：100 分鐘。
- 3.本試卷共有四題選擇題、六題計算題，一題 10 分，共計 100 分。
- 4.請將答案寫在答案卷上。
- 5.考試中禁止使用手機或其他通信設備。
- 6.考試後，請將試題卷及答案卷一併繳交。
- 7.本試卷採雙面影印，請勿漏答。
- 8.應試時不得使用電子計算機。

I. 選擇題(40%)

1. () Please identify linearity of the following differential equation and verify that the given function is the solution. (c is an arbitrary constant).

(A) $y' + \frac{x^2 - y^2}{2xy} = 0, \quad y^2 = ax - x^2$ (B) $(y')^2 - xy' + y = 0, \quad y = cx - c^2$

(C) $y' + 2y = 4(x^2 + 1)^2, \quad y = e^{3x} + 2x^2 + 2x + 1$ (D) $y'^2 = -\frac{y}{x}, \quad xy = c$

(E) $y' + 4y = 2, \quad y = ce^{-4x} + \frac{1}{2}$ (10%)單選

2. () Find the inner product $\vec{A} \cdot \vec{B}$ and the angle θ between two vectors $\vec{A} = 4\vec{i} + 3\vec{k}$ and

$$\vec{B} = -\vec{i} + \vec{j} - 1\vec{k}$$

(A) $\vec{A} \cdot \vec{B} = 5\sqrt{3}, \theta = \cos^{-1} \frac{-1}{5\sqrt{3}}$

(B) $\vec{A} \cdot \vec{B} = 1, \theta = \cos^{-1} \frac{-1}{5\sqrt{3}}$

(C) $\vec{A} \cdot \vec{B} = -1, \theta = \cos^{-1} \frac{1}{5\sqrt{3}}$

(D) $\vec{A} \cdot \vec{B} = -1, \theta = \cos^{-1} \frac{-1}{5\sqrt{3}}$

(E) $\vec{A} \cdot \vec{B} = 1, \theta = \cos^{-1} \frac{1}{5\sqrt{3}}$ (10%)單選

3. () Evaluation of determinants of
$$\begin{bmatrix} 2 & 0 & -4 & 9 \\ 4 & 5 & 1 & 0 \\ 0 & 2 & 6 & -1 \\ -3 & 8 & 9 & 1 \end{bmatrix}$$

(A) 1134 (B) 1143 (C) 1153 (D) 1135 (E) 1164 (10%)單選

4. () Please solve the differential equation: $y'' + y = \sec x$

(A) $y = c_1 \cos x + c_2 \sin x + \cos x \ln|\cos x| + x \sin x$

(B) $y = c_1 \cos x + c_2 \sin x + \cos x \ln|\sec x| + x \csc x$

(C) $y = c_1 \cos x + c_2 \sin x + \cos x \ln|\sin x| + x \cos x$

(D) $y = c_1 \cos x + c_2 \sin x + \sec x \ln|\cos x| + x \sin x$

(E) $y = c_1 \cos x + c_2 \sin x + x \ln|\cos x| + x \sin x$ (10%)單選

II. 非選擇題(60%)

1. Solve the O.D.E by Laplace transform $y'' + 4y' + 3y = 0$, $y(0) = 0$, $y'(0) = 4$ (10%)

2. Solve the differential equation: $\begin{bmatrix} y_1'(x) \\ y_2'(x) \end{bmatrix} = \begin{bmatrix} 4 & 1 \\ -1 & 2 \end{bmatrix} \begin{bmatrix} y_1(x) \\ y_2(x) \end{bmatrix}$ (10%)

3. Please find the function $y(x)$ to satisfy the differential equation

$$y'' + 5y' + 6y = 0, \quad y(0) = 0, y'(0) = 3. \quad (10\%)$$

4. Please find the function $y(x)$ to satisfy the equation and initial conditions. (10%)

$$x dy = 3y dx, \quad y(1) = 1$$

5. Solve the following D.E. in general solution by explicit solution form. (10%)

$$y' e^{\pi x} = y^2 + 1$$

6. Solve the initial value problem in particular solution. (10%)

$$xy' = y + x^2 \sec(y/x), \quad y(1) = \pi$$