

國 立 宜 蘭 大 學

1 0 3 學 年 度 研 究 所 碩 士 班 考 試 入 學

工程數學二試題

(電 機 工 程 學 系 碩 士 班)

准考證號碼：

《作答注意事項》

- 1.請先檢查准考證號碼、座位號碼及答案卷號碼是否相符。
- 2.考試時間：100 分鐘。
- 3.本試卷共有七題，共計 100 分。
- 4.請將答案寫在答案卷上。
- 5.考試中禁止使用大哥大或其他通信設備。
- 6.考試後，請將試題卷及答案卷一併繳交。
- 7.本試卷採雙面影印，請勿漏答。
- 8.應試時不得使用電子計算機。

1. Consider the binary communication channel shown in the Fig.1. The channel input symbol X may assume the state 0 or the state 1, and, similarly, the channel output symbol Y may assume either the state 0 or the state 1. Because of the channel noise, an input 0 may convert to an output 1 and vice versa. The channel is characterized by the channel transition probabilities p_0, q_0, p_1 and q_1 , defined by $p_0 = P(y_1|x_0), p_1 = P(y_0|x_1), q_0 = P(y_0|x_0)$ and $q_1 = P(y_1|x_1)$, where x_0 and x_1 denote the events $X=0$ and $X=1$, respectively, and y_0 and y_1 denote the events $Y=0$ and $Y=1$, respectively. Note that $p_0 + q_0 = 1 = p_1 + q_1$.

Let $P(x_0) = 0.5, p_0 = 0.1,$ and $p_1 = 0.2$.

- Find $P(y_0)$ and $P(y_1)$.
- If a 0 was observed at the output, what is the probability that a 0 was the input state?
- If a 1 was observed at the output, what is the probability that a 1 was the input state?
- Calculate the probability of error P_e .

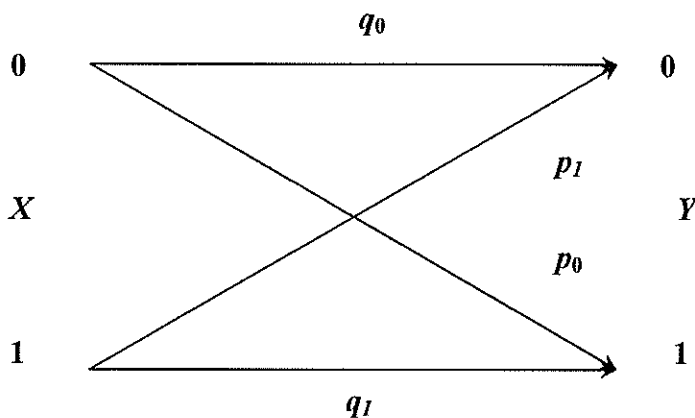


Fig. 1

(15%)

2. A mixed type(continuous and discrete) random variable X is defined by the cumulative

$$\text{distribution function (cdf) } F_X(x) = \begin{cases} 0 & x < 0 \\ \frac{1}{2}x & 0 \leq x < 1. \\ k & 1 \leq x \end{cases}$$

- Find the value of k .
- Find the probability density function(pdf) corresponding to the cdf.
- Find (i) $P(\frac{1}{2} < X \leq 1)$; (ii) $P(\frac{1}{2} < X < 1)$; and (iii) $P(X > 2)$.

(15%)

3. A random variable X is uniformly distributed in the interval $[1, 4]$. Find and sketch the probability density function(pdf) of the random variable $Y = -3X + 5$.

(10%)

(後面尚有題目)

4. A fair die is rolled. Find the expected number of spots up and the variance of the number of spots up. (15%)

5. Two random variables have joint pdf

$$f_{XY}(x, y) = \begin{cases} C, & x^2 + y^2 \leq 1, x > 0, y > 0 \\ 0, & \text{otherwise} \end{cases}$$

(a) Find the constant C .

(b) Find $E(XY)$, $E(X)$, and $E(Y)$.

(c) Are these random variables uncorrelated? (15%)

6. Let $Y = \sin(X)$, where X is uniformly distributed over $[0, 2\pi]$. Find the mean and variance of Y . (15%)

7. Let X and Y be independent random variables, each uniformly distributed over $[0, 1]$.

Let $Z = X + Y$, $W = X - Y$. Find the marginal pdf's of Z and W . (15%)