

1. Try to find the following solution

$$x^2 y'' + xy' - 4y = 0, y(1) = 1, y(0) = 0 \quad (20\%)$$

2. (a) What is the mathematical meaning of  $\nabla f$  (the gradient of function  $f(x, y)$ ).

(b) Vector A is expressed as  $x i + y j - z^2 k$ , find the divergence and curl of vector A? (20%)

3. The transfer function of a linear system is defined as the ratio of the Laplace transform of the output variable to the Laplace transform of the input variable, with all initial conditions assumed to be zero. The transfer function of a system is

$$\frac{Y(s)}{R(s)} = \frac{10(s+1)}{s^2 + 10s + 21}$$

Determine  $y(t)$  when  $r(t)$  is a unit step input. (20%)

4. 解微分方程式  $\frac{\partial^2 u}{\partial t^2} - c^2 \frac{\partial^2 u}{\partial x^2} = 0$ ，其中  $u(0, t) = u_x(1, t) = u_t(x, 0) = 0$  且  $u(x, 0) = x$  (20%)

5. 求下列週期函數  $f(t)$  之傅立葉展開式 (Fourier Expansion).

在一個週期內,

$$\begin{aligned} f(t) &= 0 & -\pi < t < 0 \\ f(t) &= \sin t & 0 < t < \pi \end{aligned} \quad (20\%)$$