

1. (13%) Find the complete solution  $y$  for the following linear differential equation

$$(x-2)^2 \frac{d^2 y}{dx^2} + 5(x-2) \frac{dy}{dx} - 5y = 0.$$

2. (12%) Find the general solution of the following equation

$$(x^2 + y^2 + 2x)dy = 2ydx.$$

3. (12%) Let the coordinate vector of  $\mathbf{x}$  with respect to the basis  $B$  be  $[\mathbf{x}]_B$ . If  $B = \{(1,1,0), (1,0,1), (1,1,1)\}$ ,  $B_2 = \{(1,0,0), (1,0,1), (1,1,1)\}$ , and  $[\mathbf{x}]_B = (1,2,3)$ ,

Find  $[\mathbf{x}]_{B_2}$ .

4. (7%) (a) Find the eigenvalues and eigenvectors for the matrix  $\mathbf{A}$

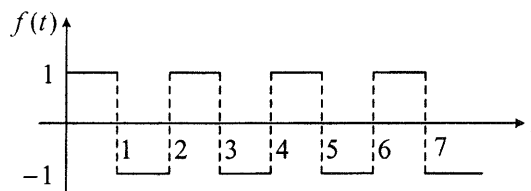
$$\mathbf{A} = \begin{bmatrix} 1 & -4 \\ 2 & -5 \end{bmatrix}.$$

(6%) (b) Find  $e^{\mathbf{A}^2}$ .

5. (14%) Use power series method to solve the following differential equation. Find the first three nonzero terms of two linearly independent Frobenius solutions.

$$\frac{d^2 y}{dx^2} - \left(\frac{1}{2x}\right) \frac{dy}{dx} + \left(\frac{1}{x}\right) y = 0$$

6. (12%) Find the Laplace transform  $L\{f(t)\}$  in the form of hyperbolic tangent function  $\tanh(\cdot)$ , where  $f(t)$  is the periodical square wave shown in the following figure.



7. (12%) Given a smooth curve  $R(t) = 3\sin(t)\vec{i} + 3\cos(t)\vec{j} + 4t\vec{k}$ , find the value of term  $w = 25(\kappa + \tau)$  where  $\kappa$  is the curvature and  $\tau$  is the torsion of the curve.

8. (12%) Evaluate  $\int_C |z|^2 dz$ , where  $C$  is the straight line segment from 1 to  $i$ .