Question 1

(1) Which one of the followings is the hexadecimal representation of 671.23_8? (5 marks)

(A) 101111001.010011	(K) 68.5
(B) 110111001.010011	(L) 1119.412
(C) 101110001.010011	(M) 119.4B
(D) 10111101.1011	(N) B9.43
(E) 11011101.1011	(O) 1B9.4C
(F) 10111001.1011	(P) 1B9.43
(G) 21321.103	(Q) 119.4C
(H) 12321.103	(R) BD.B
(I) 11301.103	(S) DA.B
(J) 671.23	(T) B9.B

(2) Use binary arithmetic to perform the calculation $10111_2 \times 1101_2$. Show all workings. (5 marks)

Question 2

(1) Use the laws of logic (at page 5) to simplify $\neg(p \vee \neg q) \vee (\neg p \wedge \neg q)$ as far as possible.
Show all workings. At each step, apply ONLY one law and write down the name of the law used. (5 marks)

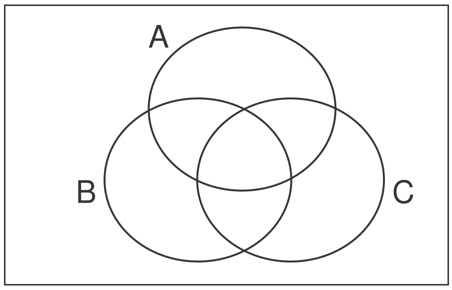
Steps	Name of the law used

(2) Complete the truth table for the expression $[p \to (p \wedge q)] \to \neg q$. (5 marks)

p	q	$p \wedge q$	$p \to (p \wedge q)$	$\neg q$	$[p \to (p \wedge q)] \to \neg q$
T	T				
T	F				
F	T				
F	F				

Question 3

(1) Suppose the universal set is {1, 2, 3, 4, 5, 6, 7, 8} and
A = {1, 2, 3, 6, 8}, B = {6, 7, 8}, C = {3, 6}.



(i) Depict the sets on a Venn diagram (as shown at the right). (3 marks)

(ii) Write the set $\overline{(A \cup C)} \cup B$ in enumerated form. (3 marks)

(iii) Write the set $B \times C$ in enumerated form. (2 marks)

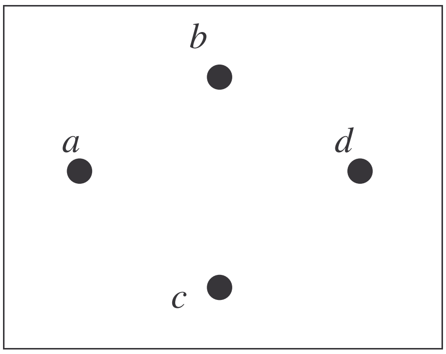
(iv) Write the set $C^2 \cap (A \times B)$ in enumerated form. (2 marks)

Question 4

Let R be the relation on {a, b, c, d} defined by the matrix as shown at the right.

	a	b	c	d
a	T	F	T	F
b	F	T	T	F
c	T	T	T	F
d	F	F	F	T

(i) Draw the graphical representation of R. (4 marks)

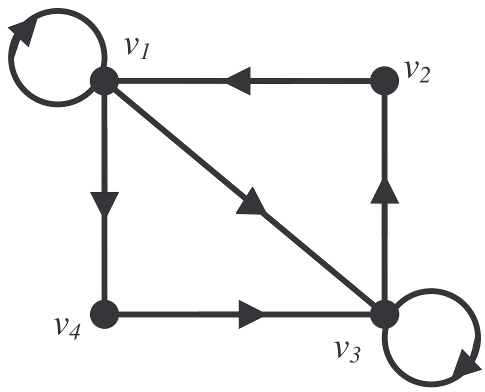


(ii) Which of the following statements are correct? (6 marks)

(A) R is reflexive
(B) R is not reflexive
(C) R is symmetric
(D) R is not symmetric
(E) R is transitive
(F) R is not transitive

Question 5

(1) Determine the adjacency matrix of the directed four-vertex graph depicted in the figure. (4 marks)

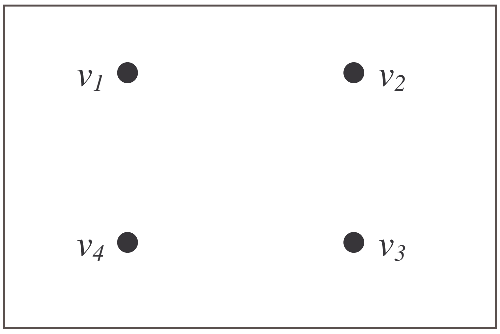


$$\begin{array}{c} v_1 \quad v_2 \quad v_3 \quad v_4 \end{array}$$

$$\begin{array}{c} v_1 \\ v_2 \\ v_3 \\ v_4 \end{array} \left[\begin{array}{cccc} \quad & \quad & \quad & \quad \\ & & & \\ & & & \\ & & & \end{array} \right]$$

(2) Let $G=(V,E)$ be a graph with vertices $V=\{v_1,v_2,v_3,v_4\}$ and with edges
$E=\{\{v_1,v_2\},\{v_2,v_3\},\{v_1,v_4\},\{v_3,v_4\},\{v_1,v_3\}\}$.

(i) Draw G. (2 marks)



(ii) List all walks of length 3 between v_1 and v_2. (2 marks)

(3) How many edges does a complete graph on n vertices have? (2 marks)

Question 6

In how many ways can $2n$ people be divided into n pairs? (10 marks)

Question 7

Let $A_X=\{1,2,3,4\}$, $P_X=\{1/2, 1/4, 1/8, 1/8\}$, and consider the code
$c(1)=0$, $c(2)=10$, $c(3)=110$, $c(4)=111$.
Please find the expected length $L(C)$ of this code. (10 marks)

Question 8

There is a 30 percent chance that it rains on any particular day.

(i) What is the probability that there is at least one rainy day within a 7-day period?　(5 marks)

(ii) What is the probability that there is at least two rainy day within a 7-day period?　(5marks)

Question 9

(1) Consider the following recursive algorithm, named *exam*.
List the output (in order) that is generated by the algorithm if $m = 3$.　(6 marks)

Algorithm *exam*(m)
　If $m = 1$ then
　　$t \leftarrow 2$
　else
　　$exam(m-1)$
　　$t \leftarrow 2t + m$
　Output t

(2) Consider the function defined recursively for all positive integers n by $f(1) = 1$ and $f(n) = f(n-1) + 2n - 1$ for $n > 1$. Find a simple formula for $f(n)$.　(4 marks)

Question 10

(1) Let $(A, *)$ be a semigroup. Show that, for a, b, c *in* A, if $a * c = c * a$ *and* $b * c = c * b$, then $(a * b) * c = c * (a * b)$.　(5 marks)

(2) Let $(A, *)$ be a commutative semigroup. Show that, for a, b *in* A, if $a * a = a$ *and* $b * b = b$, then $(a * b) * (a * b) = a * b$.　(5 marks)

Law(s)		Name
$p \leftrightarrow q \equiv (p \rightarrow q) \wedge (q \rightarrow p)$		equivalence law
$p \rightarrow q \equiv \neg p \vee q$		implication law
$\neg\neg p \equiv p$		double negation law
$p \wedge p \equiv p$	$p \vee p \equiv p$	idempotent laws
$p \wedge q \equiv q \wedge p$	$p \vee q \equiv q \vee p$	commutative laws
$(p \wedge q) \wedge r \equiv p \wedge (q \wedge r)$	$(p \vee q) \vee r \equiv p \vee (q \vee r)$	associative laws
$p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$	$p \vee (q \wedge r) \equiv (p \vee q) \wedge (p \vee r)$	distributive laws
$\neg(p \wedge q) \equiv \neg p \vee \neg q$	$\neg(p \vee q) \equiv \neg p \wedge \neg q$	de Morgan's laws
$p \wedge \mathrm{T} \equiv p$	$p \vee \mathrm{F} \equiv p$	identity laws
$p \wedge \mathrm{F} \equiv \mathrm{F}$	$p \vee \mathrm{T} \equiv \mathrm{T}$	annihilation laws
$p \wedge \neg p \equiv \mathrm{F}$	$p \vee \neg p \equiv \mathrm{T}$	inverse laws
$p \wedge (p \vee q) \equiv p$	$p \vee (p \wedge q) \equiv p$	absorption laws

Laws of logic