1.Find the general solution for each of the following differential equations.

(a). $\dfrac{dy}{dx} = \dfrac{1 + y^2 + 3x^2 y}{1 - 2xy - x^3}$ (10%)

(b). $y'' + y' - 2y = 5t + e^{2t}$, $y(0) = y'(0) = 1$ (10%)

2.Use the Laplace transforms to solve the given integral equation. (10%)

$$y(t) = 2 - 3e^{-t} - \int_0^t e^{(t-\tau)} y(\tau)\, d\tau$$

3.Use the Laplace transforms to solve the following equation. (10%)

$$y'' + y' = g(t) \quad , \quad g(t) = \begin{cases} 0 & , \quad 0 \le t \le 2 \\ 5 & , t > 2 \end{cases} \quad , \quad y(0) = y'(0) = 0$$

4.Find the inverse Laplace transforms of the following equation. (10%)

$$F(s) = \frac{s+1}{(s+2)(s^2+2s+26)}$$

5. (a). Find the Fourier series of the function $f(x)$,

where $f(x) = x + \pi$ if $-\pi < x < \pi$ and $f(x + 2\pi) = f(x)$. (10%)

(b).Using (a) to evaluate $1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + - \cdots = ?$ (5%)

6. (a). Find the eigenvalues and eigenvectors of the matrix

$$A = \begin{bmatrix} -1 & -2 \\ 3 & 4 \end{bmatrix}.$$ (10%)

(b). Using Cayley-Hamilton Theorem to evaluate A^{20}. (5%)

7. (a). Show that the function $f(z) = \tan(\frac{1}{z+1})$ there is infinitely many singularities, only one of which is nonisolated. (5%)

(b) Evaluate $\oint_C \frac{z+3}{z(z-\pi)(z-7)} dz$, the contours C consists of the circle $|z| = 6$, described in the positive direction, together with the circle $|z| = 4$, described in the negative direction.

(5%)

8. Find the Cauchy principal value of the integral $I = \int_{-\infty}^{\infty} \frac{3}{(x^2+1)(x-1)} dx$. (10%)