Part I. 單擇題（共30分，每題五分、答錯倒扣一分）

1. The general solution of $(x+1)y'-(2x+3)y=0$ is (A) $(c_1+c_2x)e^x$ (B) $c(x+1)^2e^{-x}$

   (C) $cx+x\ln x$ (D) $c(1+x)e^{2x}$ (E) $-1+cx^3$.

2. The inverse Laplace transform of the given function
   $\mathcal{L}^{-1}\left\{\dfrac{1}{s^2(s^2+4)}\right\}=a_0+a_1t+a_2t^2+b_0\cos\omega t+b_1\sin\omega t$, then (A) $\omega=4$

   (B) $a_0=1$ (C) $b_0=0$ (D) $a_1=1/8$ (E) $b_1=1/8$.

3. Let $A\in\Re^{m\times n}$, with $m<n$, and $Null(A)=Span(e_1)$. What is the rank of A? (A) n, (B) m (C) $n-1$ (D) $m-1$ (E) $n-m$

4. Let A be a 2×2 matrix, $B:2\times2$, $C:2\times3$, $D:3\times2$, and $E:3\times1$ respectively. Determine which of the following matrix expressions exist.

   (A) $3(BA)(CD)+(4A)(BC)D$ (B) A^2D (C) $DC+BA$ (D) $C-3D$ (E) B^3+3CE

5. Which of the following transformation is not a linear mapping?

   (A) $T:\Re^{m\times n}\to\Re^{n\times m},T(X)=X^T$ (B) $T:\Re^{m\times n}\to\Re,T(X)=\det(X)$

   (C) $T:\Re^{m\times n}\to\Re,T(X)=tr(X)$ (D) $d/dx:\Re[x]\to\Re[x],d/dx$ is the differential operator.

   (E) $\Re^R=\{f\mid f:\Re\to\Re\},T:\Re^R\to\Re,T(f)=f(3)$.

6. Let $A=\begin{bmatrix}2&3&0&0\\2&4&0&0\\715&245&1&2\\305&570&2&7\end{bmatrix}$, then $\det(A)=?$ (A)0 (B) 6 (C)3115 (D) -170 (E) 107.

※ 注意：請在答案卷上作答，寫在試題卷之答案不予採計。

Part II. 計算題（共70分，每題10分）

1. Solve the differential equation $(D^2 - 8D + 16)y = 8\sin 2x + 3e^{4x}$.

2. Solve P.D.E. $\dfrac{\partial u(x,y)}{\partial x} + \dfrac{\partial u(x,y)}{\partial y} = 2(x+y)u(x,y)$

3. The Periodic function $f(t)$ with $T = 2\pi$ defines as following :

$f(t) = \begin{cases} 0, & -\pi < t < 0 \\ \sin t, & 0 < t < \pi \end{cases}$, It can be represented by Fourier series:

$f(t) = \dfrac{a_0}{2} + \sum_{n=1}^{\infty}(a_n \cos nt + b_n \sin nt)$, find a_0, a_n and b_n =?

4. Solve $\begin{bmatrix} y_1' \\ y_2' \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ -2 & 4 \end{bmatrix}\begin{bmatrix} y_1 \\ y_2 \end{bmatrix}$, $y_1(0) = 2, y_2(0) = 3$.

5. Let $A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 1 & 1 \end{bmatrix}$, $b = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}$. Find (a) the least squares solution of $Ax = b$, and (b) the projection of b onto the span of A.

6. Show that $\begin{vmatrix} 1 & a & a^2 & a^3 \\ 1 & b & b^2 & b^3 \\ 1 & c & c^2 & c^3 \\ 1 & d & d^2 & d^3 \end{vmatrix} = \begin{vmatrix} 1 & b+c+d & bc+cd+db & bcd \\ 1 & c+d+a & cd+da+ac & cda \\ 1 & d+a+b & da+ab+bd & dab \\ 1 & a+b+c & ab+bc+ca & abc \end{vmatrix}$.

7. Let $A = \begin{bmatrix} 3 & 0 & -2 \\ 0 & 2 & 0 \\ -2 & 0 & 0 \end{bmatrix}$, find a diagonal matrix D and an orthogonal matrix S such that $A = SDS^{-1}$, then compute A^5.