

Part I. 單選題 (每題五分，答錯倒扣一分)

1. The general solution of $y'' + y = 0$ is (A) $c_1 \cos(x) + c_2 \sin(x)$ (B) $(c_1 + c_2 x)e^x$
(C) $c_1 e^x + c_2 e^{-x}$ (D) $c_1 x + c_2 x \ln x$ (E) $c_1 x + c_2 \ln x$

2. Among the following differential equations, which one is **exact** (A) $x^2 dy + 4y^3 dx = 0$
(B) $5y dy + x dx = 0$ (C) $y dx - 2x dy = 0$ (D) $5 \sinh(y) dx + x \cosh(y) y = 0$
(E) $x^2 e^y y' + y e^x = 0$

3. Which of the following is the Laplace transform of function t^2 (A) $\frac{1}{s^2}$ (B) $\frac{2}{s^2}$ (C) $\frac{1}{s^3}$
(D) $\frac{2}{s^3}$ (E) $\frac{1}{s}$

4. The directional derivative of $f(x, y) = x^2 + y^2$ at point (1,1) along $\vec{u} = \frac{1}{2}\vec{i} + \frac{\sqrt{3}}{2}\vec{j}$ is
(A) $\frac{2+\sqrt{3}}{2}$ (B) 0 (C) $1+\sqrt{3}$ (D) 1 (E) 2

5. Given that $A = \begin{bmatrix} 1 & 2 & 1 & 1 \\ 2 & -1 & 1 & 2 \\ 4 & 3 & 3 & 4 \\ 2 & -1 & 3 & 5 \end{bmatrix}$. The rank of A equals (A) 1 (B) 2 (C) 3 (D) 4
(E) 5

6. Which of the following matrix is Hermitian

(A) $\begin{bmatrix} 3+4i & -5i \\ -7 & 2-i \end{bmatrix}$ (B) $\begin{bmatrix} 3+i & -5 \\ -5 & 22 \end{bmatrix}$ (C) $\begin{bmatrix} 3+4i & -5i \\ -7 & 2-i \end{bmatrix}$ (D) $\begin{bmatrix} 3 & 2+i \\ 2+i & 2 \end{bmatrix}$
(E) $\begin{bmatrix} 2 & 1-5i \\ 1+5i & -4 \end{bmatrix}$

7. Let $A = \begin{bmatrix} 2 & 1 & 0 & 0 \\ 0 & 2 & 0 & 0 \\ 20 & 20 & 3 & 1 \\ 20 & 20 & 1 & 1 \end{bmatrix}$, then $\det(A) = ?$ (A) 0 (B) 2 (C) 4 (D) 8 (E) 12

8. Which of the following collections of vectors is linear dependent
- (A) $(1, 1, 1)^T, (1, 1, 0)^T, (1, 0, 0)^T$
- (B) $(1, 2, 4)^T, (2, 1, 3)^T, (4, -1, 2)^T$
- (C) $(1, 0, 1)^T, (0, 1, 0)^T$
- (D) $p_1(x) = x^2 - 2x + 3, p_2(x) = 2x^2 + x + 8, p_3(x) = x^2 + 8x + 7$
- (E) $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}, \begin{bmatrix} 0 & 1 \\ 0 & 1 \end{bmatrix}, \begin{bmatrix} 1 & 0 \\ 1 & 0 \end{bmatrix}$

Part 2. 計算題 (每題十二分)

1. Solve the differential equation $y'' - 8y' + 16y = 0, y(0) = y'(0) = 1$
2. (i) Calculate the in Laplace transform $e^{-t} - e^{-2t}$ (ii) Calculate the inverse Laplace transform for $\frac{2}{s(s+1)}$
3. A periodic function $f(t)$ with period $T = 4$ is defined as $f(t) = 1, -1 \leq t \leq 1$. This function can be represented by Fourier series: $f(t) = \frac{a_0}{2} + \sum_{n=1}^{\infty} (a_n \cos nt + b_n \sin nt)$. Find a_0, a_n and b_n
4. Let

$$A = \begin{bmatrix} 3 & 2 \\ 3 & -2 \end{bmatrix}$$

Find all the eigenvalues and the corresponding eigenspaces

5. Given the basis $\{(1, 2, -2)^T, (4, 3, 2)^T, (1, 2, 1)^T\}$ for R^3 , use the Gram-Schmidt process to obtain an orthonormal basis.